MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

O. M. BEKETOV NATIONAL UNIVERSITY of URBAN ECONOMY in KHARKIV

A. V. Bezuhlyi
N. S. Shyshko

PHYSICS
GENERAL PHYSICS

LECTURE NOTES

(for full-time and part-time students of the first (bachelor's) level of higher education in all specialties)

Kharkiv
O. M. Beketov NUUE
2025

UDC 53(075.8)

Bezuhlyi A. V. Physics. General Physics : lecture notes (for full-time and part-time students of the first (bachelor's) level of higher education in all specialties) / A. V. Bezuhlyi, N. S. Shyshko; O. M. Beketov National University of Urban Economy in Kharkiv. – Kharkiv: O. M. Beketov NUUE, 2025. – 134 p.

Authors
PhD (phys. & math.), assoc. prof. A. V. Bezuhlyi
senior lecturer N. S. Shyshko

Reviewer
O. M. Petchenko, doctor of physical and mathematical sciences, professor, professor of the department of physics, O. M. Beketov National University of Urban Economy in Kharkiv

Recommended by the department of physics, record № 3 on November 25, 2024

© A. V. Bezuhlyi, N. S. Shyshko, 2025
© O. M. Beketov NUUE, 2025

CONTENTS

FOREWORD

The rapid advancement of science across various fields has led to an increase in the number of academic disciplines and a reduction in the time available for studying physics. This necessitates new approaches to the educational process. Consequently, there is a need for a concise summary that aligns with the scope of university-level physics courses to provide an initial introduction to the subject.

This course includes lectures covering nearly all the material in the curriculum in a condensed form. The mathematical framework is kept to a minimum, with greater emphasis placed on the physical meaning of concepts, phenomena, and laws. This approach enables students to grasp the fundamental concepts, phenomena, and laws of physics within a limited timeframe.

The International System of Units (SI) is used consistently throughout the summary.

PART 1

THE PHYSICAL FUNDAMENTALS OF MECHANICS, MOLECULAR PHYSICS AND ELECTROSTATICS

CHAPTER 1

MECHANICS

1.1 Mechanical Motion

Mechanics is a branch of physics studying the simplest forms of motion. It consists of the movement of bodies in the space relative to one another. To describe quantitatively the motion of the body, it is necessary to introduce a reference frame – select a body of reference and link it with the coordinate system and timepiece to determine the time.

Mechanics is divided into three sections – kinematics, dynamics and statics.

1.2 Kinematics of a particle point

Kinematics studies the motion without considering forces acting on the body.

In kinematics on create approximate models, neglecting some irrelevant factors. For example, in the study of elementary – translational motion on introduce the concept of a *point particle* (or simply a particle). Particle is the body, the size of which can be neglected in the given problem. *Translational* is called the motion of the body all the points which move so that the line connecting the two any points of the body remains in parallel to itself. Then, to describe the

motion of the body is sufficient to describe the movement of one point anywhere in the body.

Vector position. Position of a point in space can be characterized by the vector $\vec{r}$ drawn from the origin of coordinates to given point which direction is defined by three orths (units \ vectors) $\vec{i}, \vec{j}, \vec{k}$:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} .$$

In this

$$r = \sqrt{x^2 + y^2 + z^2} , \qquad (1.1)$$

$r_x = x$; $r_y = y$; $r_z = z$, $\Delta\vec{r}$ – vector of small displacement for a small interval of time Δt, ΔS – is the length of the trajectory. Displacement vector $\Delta\vec{r}$ is directed from the initial point at the moment time t to the final point at the moment $t+\Delta t$ (Fig. 1.1).

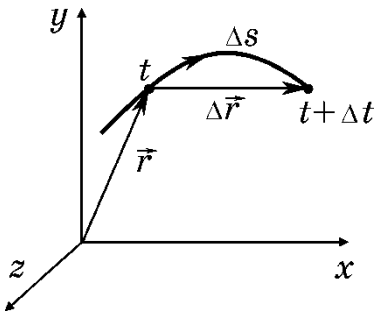


Figure 1.1 – Particle displacement Vector

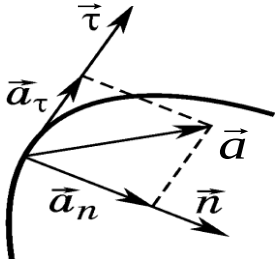


Figure 1.2 – Acceleration Vector Decomposition

Speed. In uniform motion body goes in equal intervals of time equal ways, i.e., speed is determined by the formula

$$v = \frac{S}{t}. \qquad (1.2)$$

The average value of the speed of uneven movement, by definition, is scalar:

$$v = \frac{\Delta S}{\Delta t} \ . \qquad (1.3)$$

Module instantaneous velocity is the first derivative of the traveling path with respect to time

$$v = \lim_{\Delta t \to 0} \frac{|\Delta S|}{\Delta t} = \frac{dS}{dt}. \qquad (1.4)$$

Vector instantaneous velocity is the first derivative of vector position with respect to time and directed along a tangent to the trajectory.

Projections of velocity onto the coordinate axes are defined by the formulas

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt}.$$

Calculation of distance. To determine the traveled path in uneven motion, it must break all the way to the parts, so small that the velocity at each elementary path can be considered constant. Then using formula (1.4), we obtain

$$S = \sum_i \Delta S_i \approx \sum_i v_i \Delta t_i. \qquad (1.5)$$

Formula (1.5) is more precisely, the smaller intervals Δt, ΔS. Strict equality signs in (1.5) can be placed just under the sign of limit:

$$S = \lim_{\Delta t_i \to 0} \sum_i V_i \Delta t_i = \int V dt. \qquad (1.6)$$

Acceleration. If $\vec{v}$ is the increment speed during the time Δt, the average acceleration is defined by the formula

$$a_{avg} = \Delta v \Delta t. \qquad (1.7)$$

Instant acceleration is the first time derivative of the velocity with respect to time

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}. \qquad (1.8)$$

Acceleration characterizes the change velocity per unit of time. In general, the curvilinear motion vector acceleration

decomposed into two components – tangential component, directed along the tangent and normal component, directed along the perpendicular to the tangent (see Fig. 1.2):

$$\vec{a} = \overrightarrow{a_\tau} + \overrightarrow{a_n}, \quad (1.9)$$

where

$$a_\tau = \frac{dv}{dt}; \quad (1.10)$$

$$a_n = \frac{v^2}{R}; \quad (1.11)$$

R – radius of curvature of the trajectory at the given point. Tangential component (tangential acceleration) determines the change of velocity per unit time in magnitude, and the normal component (normal acceleration) – in direction. Then the magnitude of full acceleration

$$a = \sqrt{a_\tau^2 + a_n^2}.$$

Calculation speed. For uniform accelerated ($\vec{a} = const$ movement at regular intervals Δt velocity changes by the same magnitude Δv. Therefore, the movement acceleration

$$\vec{a} = \frac{\vec{v} - \overrightarrow{v_0}}{\Delta t}, \quad (1.12)$$

$\overrightarrow{v_0}$ – initial velocity, i.e. the velocity at t=0.

To determine in general the case of velocity change in time t, we will divide all the time movement on the intervals so small that the acceleration $\vec{a}_i$ at each site can will be able to consider constant for each period Δt. Then, use the formula (1.12) we obtain

$$\vec{v} - \overrightarrow{v_0} = \sum_i \Delta \overrightarrow{v_i} \approx \sum_i \vec{a}_i \Delta t_i. \quad (1.13)$$

Formula (1.13) more precisely, the smaller the gap. Strongly equal sign can be placed just under the sign of the limit, i.e.

$$\vec{v} - \overrightarrow{v_0} = \lim_{\Delta t \to 0} \sum_i \overrightarrow{a_i} \Delta t_i, \quad (1.14)$$

or

$$\vec{v} = \vec{v}_0 + \int_0^t \vec{a}(t)\,dt. \qquad (1.15)$$

At $\vec{a}$ =*const* formula (1.15) becomes known formula for uniform movement

$$\vec{v} = \overrightarrow{v_0} + \vec{a}t \qquad (1.16)$$

or in scalar form

$$v = v_0 \pm at, \qquad (1.17)$$

where the plus sign corresponds to uniformly accelerated motion, and a minus sign – to uniformly reduced motion.

Substituting (1.17) into (1.6), we obtain the known formula for determination the way in uniformly accelerated motion:

$$S = v_0 t \pm \frac{at^2}{2}. \qquad (1.18)$$

1.3 Dynamics of a point particle

Dynamics studies the motion of bodies in connection with its causes (owing to the forces acting on the body) resulting in the occurrence of a specific change of motion. The primary concepts dynamics are mass and force. Mass is a measure of inertia of the body. Reason for the change of movement is the force. Force is determined the magnitude and direction of action one body to another, and thus is a measure of the interaction between the bodies. Force is characterized, by module, direction and a point of application.

Momentum of a particle is the product of its mass and velocity:

$$\vec{p} = m\vec{v}. \qquad (1.19)$$

momentum of a system particles is the vector sum of momentums of particles that make up the system:

$$\vec{p}_{sys} = \vec{p}_1 + \vec{p}_2 + \cdots + \vec{p}_N = \sum_{i=1}^{N} \vec{p}_i. \qquad (1.20)$$

Newton's First law (law of inertia). The body maintains a state of rest or uniform rectilinear motion until external action (force) does not withdraw it from this state.

Reference frame, in which this law is implemented are called *inertial reference frame.* There are an infinite number of inertial frames. Any frame that moves uniformly and rectilinearly relative inertial, is also inertial.

Newton's second law (law of momentum). Rate of change of the momentum is equal to the force acting on the body:

$$\vec{F} = \frac{d\vec{p}}{dt}, \tag{1.21}$$

where F – resultant of all the forces applied to the body. For the body with constant mass, considering that the resultant is the vector sum of all forces acting on the body, the equation (1.21) is written as

$$\vec{a} = \frac{\sum \vec{F}}{m} \tag{1.22}$$

The acceleration of the body directly proportional to resulting all the forces acting on the body and inversely proportional to its mass.

Newton's third law. The forces exerted by interacting bodies on each other are equal in magnitude and opposite in direction:

$$\vec{F}_{12} = -\vec{F}_{21},$$

where first index indicates the body to which force is applied, the second – the acting body.

Equation of the center of mass (or center of inertia). Center of mass – is the point C, whose position in space is defined by the radius vector according to the formula

$$\vec{r}_c = \frac{\sum m_i \cdot \vec{r}_i}{\sum m_i} \tag{1.23}$$

where m_i – mass of point particle, $\vec{r}_i$ _ radius vector determining the position of i-th particle. Then, according to (1.20) the momentum of the system of point particles:

$$p = mv, \quad (1.24)$$

where m – mass of the entire system, v – its speed. Such a manner from (1.20), taking into account (1.24) we obtain the equation of motion of the center of mass

$$\vec{F} = m\vec{a}. \quad (1.25)$$

Law of momentum conservation. The momentum of a closed system of particles over time remains unchanged*:*

$$\vec{p} = m\vec{v} = const. \quad (1.26)$$

System of bodies which interact with on to another and doesn't interact with another bodies are called closed one.

1.4 Work and energy

Kinetic energy theorem. Work. The equation of motion of the particle is

$$m\frac{d\vec{v}}{dt} = \vec{F}. \quad (1.27)$$

Multiplying Eq. (1.27) by the elementary displacement of the particle $d\vec{S} = \vec{V}dt$, we get

$$m\vec{v}dv = \vec{F}d\vec{S}. \quad (1.28)$$

Integrating both sides of equation (1.28), we obtain the result

$$\frac{mv_1^2}{2} - \frac{mv_2^2}{2} = \int_1^2 \vec{F}\, d\vec{S}. \quad (1.29)$$

The scalar product of force vector $\overline{F}$ and elementary displacement $d\vec{S}$ – is an elementary work

$$dA = \vec{F}d\vec{S} = FdS\cos\alpha = F_s dS, \quad (1.30)$$

where $F_S = F\cos\alpha$ is projection of force on the direction of movement (i.e. the direction of instantaneous velocity vector); α – the angle between the vectors of force and displacement.

The quantity

$$A = \int_1^2 \vec{F} d\vec{S} = \int_1^2 F_s dS \qquad (1.31)$$

is the work over path 1-2.

If the force has the same magnitude on all the way, then

$$A = \vec{F}\vec{S} = F_s S\,. \qquad (1.32)$$

The left side of equation (1.29) is the difference of values of kinetic energies respectively at points 1 and 2, and the equation is a mathematical formulation of the theorem of kinetic energy: the work of the resultant of all the forces acting on the particle goes to augmentation its kinetic energy:

$$A_{12} = E_{k_2} - E_{k_1}, \qquad E_k = \frac{mv^2}{2}.$$

Power. The work done in unit time is called power. If the work ΔA is done during the time Δt, then the average power is

$$N_{avg} = \frac{\Delta A}{\Delta t}. \qquad (1.33)$$

Instantaneous power

$$N = \lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} = \frac{dA}{dt},$$

where $dA = \vec{F} d\vec{S}$.

Then for the instantaneous power we obtain the formula

$$N = \vec{F}\vec{V},$$

where the right side is the scalar product of vectors force and velocity.

Potential field forces. Potential energy. Find potential energy particles, which is in the field of *central forces*. Central forces depend only on the distance between the interacting particles and directed along the line connecting the particles. Example of such force is the gravitational force created by the gravitational field. The gravitational force of interaction between two masses m_1 and m_2 is determined by the formula

$$\vec{F}_{12} = G\frac{m_1 m_2}{r^2}\cdot\frac{\vec{r}}{|r|}, \qquad (1.34)$$

where $G = 6.67 \cdot 10^{-11}\,\frac{N\cdot m^2}{kg^2}$ – gravitational constant.

Let us calculate the work of the gravitational force on the particle mass m_1 between any two points 1 and 2 in the gravitational field of mass m_2:

$$A_{12} = \int_1^2 \vec{F}d\vec{r} = -Gm_1m_2\int_1^2 \frac{\vec{r}}{r^3}\,d\vec{r} = -Gm_1m_2\int_1^2 \frac{dr}{r^2} = -G\left(\frac{m_1m_2}{r_1}\right) + G\left(\frac{m_1m_2}{r_2}\right).$$

In this formula it take into account that $\vec{r}d\vec{r} = rdr$. From the resulting formula, it follows directly that the work of a particle moving in the gravitational field does not depend on the form of the path and depends only on its initial and final position. The value which is determined by the formula

$$E_p(r) = -G\frac{m_1m_2}{r} \qquad (1.35)$$

called *potential energy*. If the body is in space the force which continuously varies from point to point, then we say that the body is in a *force field*.

Field force, whose work does not depend on the form of the path, is called *potential* and forces acting in this field – *conservative*. Fundamentals feature potential field force – work of conservative forces along a closed path is zero.

Work conservative forces of the system are equal change of potential energy of the system

$$A_{12}^{c.f.} = E_{p1} - E_{p2}.$$

Kinetic energy is characteristic of one body in motion, and the potential-energy is characteristic of the whole system of interacting bodies.

Complete mechanical energy of the system is the sum of the kinetic energies of all bodies system and its potential energy as a whole:

$$E = \sum_{i=1}^{N} E_{k_i} + E_p .$$

The law of conservation of total mechanical energy. Changing of the total energy of the system is the work of non-conservative forces done on the: system

$$\Delta E = E_2 - E_1 = A_{12n.}.$$

If the system of bodies is located in a field only conservative forces, then $A_{12n} = 0$, and $\Delta E = 0$. Thus, for the system of bodies, which undergoes of action only conservative forces, the total mechanical energy remains constant.

The relationship between potential energy and the force given by formula

$$\vec{F} = -\nabla E_P, \qquad (1.36)$$

where $\nabla = -\left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)$ – *nabla operator*, expressions $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ means *partial derivatives*, from function that depends on several variables *(x, y, z).* For example, when the derivative $\frac{\partial}{\partial x}$ is calculated, it must to assume *y=const, z=const*. when the derivative $\frac{\partial}{\partial y}$ is calculated, it must to assume *x = const, z = const* and i.e. Field strength in potential field force is always directed toward the quickest decrease of potential energy.

1.5 Kinematics of rotational motion

Rotational motion is called motion, during which all points of the rigid body move along circles whose centers lie on a straight line, called the axis of rotation. In the study of rotational motion is assumed that during the motion a rigid

body is not deformed. This body has the name a *perfectly rigid body.*

Angular velocity. The turn of a body can be represented by vector rotation, directed along the axis of rotation. Long this vector is equal to the angle, and the direction is determined by the rule of the right screw. (Fig. 1.3).

Instantaneous angular velocity is

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\varphi}}{\Delta t} = \frac{d\vec{\varphi}}{dt}. \qquad (1.37)$$

Figure 1.3 – Rotation vector $\Delta\vec{\varphi}$

When $\vec{\omega} = const$ rotational motion is uniform, for which the following values are entered: T – orbital period. The number of revolutions in unit time ν is evidently equal to $\nu = \frac{1}{T}$ .Then the angular velocity

$$\omega = 2\pi\nu = \frac{2\pi}{T}. \qquad (1.38)$$

Angular acceleration. If for the time Δt , increment of the angular velocity is $\Delta\vec{\omega}$, and the instantaneous angular acceleration

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}. \qquad (1.39)$$

Vector angular acceleration characterizes the change in angular velocity per unit time. Vector can be changed as by changing the speed of rotation, and by rotation of the axis of rotation in space. If the direction of the axis of rotation in space is non-variable, the angular acceleration module

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}. \qquad (1.40)$$

The formula (1.40) α – projection of the vector $\vec{\alpha}$, i.e. the algebraic variable (can have different signs).

When $\alpha > 0$ vectors $\vec{\omega}$ and $\vec{\alpha}$ have the same direction – the rotation is accelerated, when $\alpha < 0$ vectors $\vec{\omega}$ and $\vec{\alpha}$ have opposite directions – the rotation is decelerated. .

Relation between linear and angular velocities and accelerations. The angle $d\varphi$ corresponds to the length of arc *dS*. Calculating the first derivative of the way with respect to time, we obtain,

$$\frac{dS}{dt} = R\frac{d\varphi}{dt}, \text{ or } v = \omega R, \qquad (1.41)$$

where R-radius of circle along which a particle is moved.

The magnitude of linear acceleration from (1.41) equal

$$\frac{dv}{dt} = R\frac{d\omega}{dt}, \text{ or } a_\tau = \alpha R.$$

1.6 Dynamics of rotational motion of a rigid body

Force moment relative to the point. The pseudo vector

$$\vec{M} = \vec{r} \times \vec{F} \qquad (1.42)$$

is called the *force moment* relative to the point *O* (or the *torque* relative to this point) from which the vector position ***r*** is drawn to the point of application of the force (see Fig.1.4).

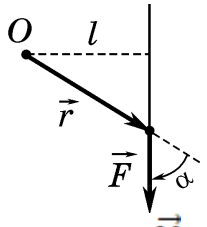


Figure 1.4 – Force moment $\vec{M}$ relative to a point

Inspection of the Fig. 1.5 shows that magnitude of the moment of force can be written in the form

$$M = Fl,$$

where $l = r\ sin\alpha$ –is the arm of the force.

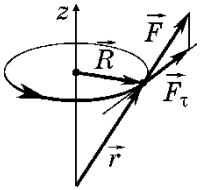


Figure 1.5 – Force moment M_z Relative to an axis *z*

Force moment relative to the axis. The force moment relative to the axis Z it is scalar whose value is determined by projection onto the given axis of the force moment vector relative to the arbitrary point of this axis

$$M_z = (\vec{r} \times \vec{F})_z , \qquad (1.43)$$

where $\vec{r}$ – vector position drawn from point of the axis to the point of application of the force $\vec{F}$. On can to rewrite the formula (1.43) as follows

$$M_z = \left|\vec{R} \times \vec{F}_\tau\right| = RF_\tau , \qquad (1.44)$$

where $\vec{F}_\tau$ – tangential component of force $\vec{F}$ (Fig. 1.5).

Inertia moment of the body. Inertia moment of the particle is called the product her mass by square of the distance ***r*** from the turn axis:

$$I = mR^2 . \qquad (1.45)$$

To determine inertia moment of the rigid body it must to divided it by elementary mass Δm_i. Then utilizing the formula (1.45) to determine inertia moment every mass Δm_i and to take the sum of the whole elementary inertia moments

$$I \cong \sum_i \Delta m_i r_i^2 . \qquad (1.46)$$

The precision of the formula (1.46) depends on the magnitude of Δm_i. Their accuracy grows with diminishing elementary mass Δm_i. The sign of equality it possible to stand in (1.46) tending $\Delta m_i \to 0$,

$$I = \lim_{\Delta m_i \to 0} \sum_i \Delta m_i \, r_i^2 = \int_M r^2 dm. \qquad (1.47)$$

Integration in the formula (1.47) is make up by the whole mass of the body *M*.

Introducing the local density

$$\rho = \lim_{\Delta m \to 0} \frac{\Delta m}{\Delta V} = \frac{d\,m}{d\,V}, \qquad (1.48)$$

integral (1.47) may be transformed in the integral over the entire volume of the body

$$I = \int_V \rho r^2 \, dV. \qquad (1.49)$$

For homogeneous body

$$I = \rho \int_V r^2 \, dV. \qquad (1.50)$$

The formula (1.50) gives a possibility to calculate the moment of inertia a body of regular form. For example, for homogenous cylinder of the radius *R* and mass *m* inertia moment relative to axis passing throw mass center equals

$$I = \frac{1}{2} mR^2. \qquad (1.51)$$

Steiner theorem. Inertia moment *I* relative to un arbitrary axis equals inertia moment I_C relative to un axis parallel to the given one and passing through the body`s center of mass plus the product of the body`s mass *m* and the square of the distance *a* between the axis:

$$I = I_c + ma^2 \qquad (1.52)$$

Fundamental equation of the dynamics of the rotational motion is formulated as follows: the resulting moment of the forces *M* acting on the body equal to the product of inertia moment of the body *I* and angular acceleration $\vec{\alpha}$

$$\vec{M} = I\vec{a}, \qquad (1.53)$$

Angular momentum of a particle. For a separate particle, the angular momentum relative to point *O* (see the Fig. 1.6) is defined as pseudo vector

$$\vec{L} = \vec{r} \times \vec{p}, \qquad (1.54)$$

where $\vec{p}$ – the momentum, $\vec{r}$ – position vector drawn from the center of rotation to the particle perpendicularly to the axis of rotation. Magnitude of the momentum can be written in the form

$$L = pl, \qquad (1.55)$$

where $l = r \sin \alpha$ – is the arm of the angular momentum.

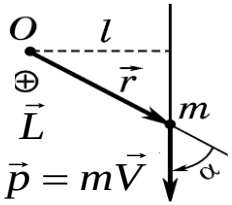


Figure 1.6 – Angular momentum of a particle relative to a point

Angular momentum relative to axis of turn Z

$$L_Z = (\vec{r} \times \vec{p})_Z \qquad (1.56)$$

where $\vec{r}$ – vector position, drawing from the center of rotation to the particle.

Angular momentum of a system particles

$$\vec{L}_s = \vec{L}_1 + \vec{L}_2 + \cdots + \vec{L}_N = \sum_{i=1}^{N} \vec{L}_i \qquad (1.57)$$

Law of conservation of angular momentum. For separate particle

$$\frac{d\vec{L}}{dt} = \vec{M}, \qquad (1.58)$$

where $\vec{M}$ – the resulting moment of the external forces applied to the particle.

For an isolated system

$$\frac{d\vec{L}}{dt} = 0 \quad \text{and} \quad \vec{L} = const.$$

The total angular momentum of an isolating system remains constant.

Angular momentum of a rigid body. For a homogeneous body rotating about an axis of symmetry it`s angular momentum *L* is equal to product of the inertia moment *I* by angular velocity-$\vec{\omega}$:

$$\vec{L} = I\vec{\omega} \tag{1.59}$$

Calculating the derivation of angular momentum (1.59) on has

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(I\vec{\omega}) = \vec{M}, \tag{1.60}$$

If $\vec{M} = 0$ then $I\vec{\omega} = const$. Hence, angular momentum $I\vec{\omega}$ remains constant, if the resulting moment of the external forces applied to the body $\vec{M}$ equals serous

1.7 Energy and work at the rotate motion

Kinetics energy. At the rotational motion about fixed axis kinetics energy is determinate by the formula

$$E_k = \frac{I\omega^2}{2}, \tag{1.61}$$

where *I* – inertia moment of body about done axis, $\vec{\omega}$ – angular velocity.

Kinetics energy of a body in plane motion. In general case the motion of body is determinate by superposition of tow motions – translational with velocity of the inertia center with $\vec{V}_c$ and rotational – with angular velocity $\vec{\omega}$ about axis passing throw the inertia center of body.

Thus, in this case on have

$$E_k = \frac{mV_c^2}{2} + \frac{I_c\omega^2}{2}, \tag{1.62}$$

where I_c – moment of inertia of the body relative to axis passing throw the inertia center of body. In the formula (1.62), the first part of the sum is the kinetic energy of translational motion of the center of mass, and the second is the kinetic energy of rotational motion relative to the axis passing through the center of mass.

Work at the rotational motion of the rigid body. Elementary work of external forces acting on the body

$$dA = M_z d\varphi, \tag{1.63}$$

where M_z - is the projection of resulting force . If M_z is the function of coordinates the work for a finite angle of turn

$$A = \int_{\varphi_1}^{\varphi_2} M d\varphi, \tag{1.64}$$

where φ_1 – initial, φ_2 – final angles of turn.

If the resulting force moment $M_z = const.$ on have

$$A = M_z \varphi. \tag{1.65}$$

1.8 Equations of motion and conditions of equilibrium of a rigid body

Equations of motion of a body:

$$m\vec{a}_c = \sum \vec{F}_i, \tag{1.66}$$

$$I\vec{a} = \sum \vec{M}_i. \tag{1.67}$$

Equation (1.66) describes the translational motion of the center of mass, which is known as Newton`s second law. Equation (1.67) describes the rotational motion of the rigid body relative to any point.

Conditions of equilibrium of a body have the form

$$\sum \vec{F}_i = 0, \quad \sum \vec{M}_i = 0. \tag{1.68}$$

1.9 The special relativity theory

According **to Galileo`s relativity principle** all mechanical phenomena in different inertial reference frames proceed identically owing to which no mechanical experiments allow us to determine whether the given reference frame is et rest or is moving uniformly in a straight line.

Let us consider two reference frames moving relative to each other with the constant velocity $\vec{V}_0$. One of these frames, designated in Fig.1.7 by the letter *K*, conditionally be considered fixed. Therefore, the second frame *K′* will move uniformly along axis z. Consequently the coordinate axis *x*, *y*, *z* of frame *K* and the axis x', y', z'. are parallel to each other.

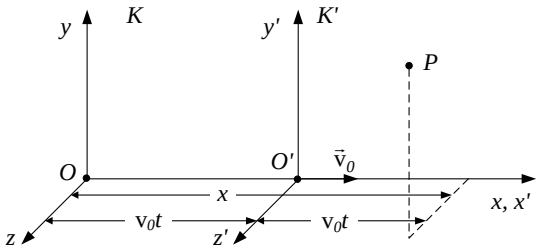


Figure 1.7 – Coordinate transformation for transition from one inertial frame of reference to another

Let us find the relation between the coordinates *x*, *y*, *z* of a point P in frame *K* and the coordinates $x' \cdot, y', z'$ of the same point in frame *K′*. If we begin to count the time from the moment when the origins of the coordinates of the tow frames coincided, then as follow Fig. 1.7 we have $x = x' + V_0 \cdot t$, $y = y'$, $z = z'$. Adding to these relations the assumption adopted in classical mechanics that the time flows the same in both frames, i.e. that $t = t'$, we get a group of four equations:

$$x = x' + V_0 \cdot t, \; y = y', \; z = z', \; t = t'. \qquad (1.69)$$

They are called Galilean transformations from frame K to frame K'. Differentiating equations (1.69) with respect to time, we find the relation between the velocity components of point P relative to the frames K and K'.

$$V_x = V_x' + V_0 \quad V_y = V_y', \; V_z = V_z'. \tag{1.70}$$

Three scalar relations are equivalent the next vector relation

$$\vec{V} = \vec{V}' + \vec{V}_0. \tag{1.71}$$

It Is the classical law off addition of the velocities.

Lorentz Transformations. In the beginning of the 20-th century the researches of the properties of electromagnetic waves leaded to the formulation of the Maxwell`s equations. As it was found, Maxwell`s equations were not invariant with respect to Galilean transformations, i.e., their form was changed for the transition from one inertial reference frame to another. The attempts to change the Maxwell`s equations so that to satisfy to Galilean transformations leaded to appearance into them additional parts, which conditioned new effects. But experiments did not confirm any new effects.

Lorentz proposed the transformations keeping the Maxwell`s equations invariable at the transition from one inertial reference frame to another:

$$x' = (x - ut)\gamma, \quad y' = y, \; z' = z, \; t' = (t - ux/c^2)\gamma, \tag{1.72}$$

where $\gamma = 1/\sqrt{1 - u^2/c^2}$;

x, y, z, t – the coordinates and time in the frame K;

x', y', z', t' – the coordinates and time in the frame K';

u – velocity the frame K' relative the frame K;

c – velocity of light in vacuum.

Corollaries of the Lorentz Transformations:

1) Examination of equations (1.72) shows that if the events occur in different points of space in the frame K, then they will not simultaneous in the frame K';

2) The length a body measured in a frame relative to which it is moving is shorter than its measured in the frame relative to which it is a rest.

Relativistic law of the velocity transformation. From the transformations (1.72) according to determination

$$V_x = \frac{dx}{dt} = \frac{v'_x + u}{1 + \frac{uv'_x}{c^2}}. \qquad (1.73)$$

Postulates of the special theory of relativity (STR). The foundation of this theory is formed by tow postulations called Einstein`s principle of relativity and the principle of constancy of the speed of light:

1) the speed of light in a vacuum is the same in all inertial reference frames and does not depend on the motion of the sources and receivers of light;

2) all laws of nature are the same in all inertial reference frames.

Relativistic expression for the momentum. To find the relativistic expression for momentum , let us changed the mass of particle *m* that moves with the velocity *u* by expression $m = m_0\gamma$. The mass m_0, the mass of motionless body is called the *rest mass,* m the mass of the body in motion – relativistic mass.

Then relativistic momentum and the equation of relativistic dynamics writes in form

$$\vec{p} = m_0\vec{u} \Big/ \sqrt{1 - u^2/c^2} \quad , \quad \vec{F} = \frac{d\vec{p}}{dt} \ . \qquad (1.74)$$

Inspecting the first of the equations (1.74) we can to conclude that at the small velocities the momentum increases owing to increasing of the numerator, because the value u^2/c^2 is very small. But when $u \to c$ the numerator increases slowly and the momentum increases owing to decrease of the denominator which tends to zeros. Therefore, at $u \to c$ to

change a little the magnitude of momentum is necessary to expand tremendous quantity of energy. Thus it is impossible to give the speed of light to a body of finite mass.

It must note that formulas (1.72) – (1.74) coincide with corresponding expressions of classical mechanics when $u \to c$.

Relativistic expression for the energy. Relation between mass and energy.

The total energy of a free particle equals to

$$E = m_0 c^2 \Big/ \sqrt{1 - u^2/c^2} \,. \qquad (1.75)$$

When $u=0$, eq. (1.75) transforms into

$$E_0 = m_0 c^2 \,.$$

This is why energy E_0 is called the *rest energy*.

It can be seen from equation (1.75) that the energy of the body and its relativistic mass are always proportional to each other. Any change in the energy of a body ΔE is attended by a change in the relativistic mass $\Delta m = \dfrac{\Delta E}{c^2}$, and, conversely, any change in the relativistic mass Δm is attended by a change in the energy of the body

$$\Delta E = \Delta m c^2 \,. \qquad (1.76)$$

This statement is called **the law of the relation between relativistic mass and energy.**

CHAPTER 2

MOLECULAR PHYSICS AND THERMODYNAMICS

2.1 Principal notions

Molecular physics is a branch of physics studying the structure and properties of a substance on the basis of the so – called molecular kinetic notions. According to these notions, any body – solid, liquid, or gaseous – consists of an enormous number of small separate particles – molecules (atoms). The molecules (atoms) are in disordered, chaotic motion. The molecules (atoms) interact between them.

The object molecular kinetic theory is to interpret the properties of bodies as the summary result of the action of molecules. It uses the statistical method and is interested only in average quantities characterizing the state of the system consisting of an enormous number of molecules.

Thermodynamics studies macroscopic properties of bodies without being interested in their microscopic picture. Thermodynamics is founded on several fundamental laws established as a result of generalizing a large amount of experimental facts.

Parameters of states. Any system can be in different states distinguished by their temperature, pressure, volume. Such quantities characterizing the state of the system are called parameters of states.

Equilibrium state of a system is meant a state in which all parameters of the system have definite values.

A process consisting of a continuous sequence of equilibrium states is called an equilibrium**.** Only an infinitely slow process can be an equilibrium one.

2.2 Ideal gas equation of state an ideal gas

The state of a given mass of a gas is determined by the values of three parameters: the pressure *p*, volume *V*, temperature by **thermodynamic temperature scale T**. The temperature *T* measured according to this scale is related to the temperature *t* according to the Celsius scale by equation

$$T = 273 + t\ ^0\text{C}$$

The unit of absolute temperature is the Kelvin (K).

Three parameters *p, V, T* are called parameters of state. These parameters are related by equation of state

$$pV = \frac{m}{\mu} RT, \qquad (2.1)$$

where R= 8.31 $\frac{J}{mol \cdot K}$ – *molar gas constant*, m – mass of gas, μ – molar mass. Molar mass equals product of Avogadro constant N_A and mass of molecule m_0

$$\mu = N_A m_0$$

where $N_A = 6.023 \times 10^{23}\ mol^{-1}$.

Equation (2.1) can be given an another form. For this purpose, we shall use the quantity –Boltzmann constant

$$k = \frac{R}{N_A} = 1.3810^{-23}\ J / K \cdot$$

Then equation (2.1) can be write in the form

$$pV = \frac{m}{\mu} N_A kT = NkT = \frac{N}{V} kT,$$

and finally

$$p = nkT, \qquad (2.2)$$

where $n=N/V$ – the concentration of molecules.

2.3 Molecular-kinetic interpretation of temperature and pressure

Gas pressure on the vessel wall in which it is located, caused by the blows of molecules. With each stroke of the molecule on the wall the momentum of molecule changes, and in according to the second and third Newton's laws force acting on the vessel wall arises. The ratio of the force acting on the wall causing by the molecules to the area of the wall determines the amount of gas pressure.

According to the basic equation of the molecular-kinetic theory ideal gas:

$$p = \frac{2}{3}\langle E_k \rangle, \tag{2.3}$$

where *p*- pressure, *n* – concentration of gas molecules, $\langle E_k \rangle$ – the average kinetic energy of the gas molecules.

This equation (2.3) establishes the relationship between the microscopic quantities (mass and speed of molecules) and macroscopic quantities – pressure.

Likening the ideal gas equation of state (2.2) and the main equation molecular-kinetic theory (2.3), we get

$$\langle E_k \rangle = \frac{3}{2} kT. \tag{2.4}$$

According to expression (2.4) the average kinetic energy of translational motion of molecules of an ideal gas is directly proportional to the absolute temperature. That temperature is a measure of the average kinetic energy of the translational of molecules of the ideal gas in the state of thermodynamic equilibrium.

2.4 Internal energy of the system. The work, quantity of heat

The internal energy of the system – is the energy of motion and interaction between particles that make up the

system: kinetic energy of thermal motion of the particles, the potential energy of their interaction, the kinetic and potential energy of the vibrational motion of atoms and molecules. The change in internal energy ΔU in the transition system with initial state with energy U_1 to the final state with energy U_2 is $\Delta U = U_2 - U_1$, and does not depend on the processes that led to the transition. If the system returns to its previous state, its internal energy remains unchanged. The internal energy of the system can change of two-ways: the performance of the work A on the body, or giving it the amount of heat ΔQ. Let into the cylinder under the piston area *S* there is gas pressure of which equals to *p*, then the pressure, force $F = pS$. If the piston is at quasi-static gas expansion is moved through the distance dx, then elementary work of gas iquals

$$dA = Fdx = pSdx = pdV.$$

The work done by the system in a finite volume expansion is equal

$$A = \int_{V_1}^{V_2} pdV. \qquad (2.5)$$

The second method of transfer of the energy is the process of heat transfer that occurs at constant external parameters.

The first law of thermodynamics is a generalization of the law of preservation of energy conversion and for thermodynamic systems

$$Q = (U_2 - U_1) + A. \qquad (2.6)$$

The amount of heat imparted to a system is spent on an increase of the internal energy $\Delta U=(U_2 - U_1)$ and on the work done by the system on external bodies *A*.

Number of degrees of freedom of a mechanical system is called the number of independent variables (to be set) by means of which we can set the position of the system in the space.

Material point has three degrees of freedom – coordinates (*x, y, z*). Solid body has six degrees of freedom:

three coordinates of the center of inertia (x_c, y_c, z_c) and three angles (θ, φ, ψ) that define the position in the space of two mutually perpendicular axes connected with the body and passing through the center of inertia (see Fig. 2.1, a).

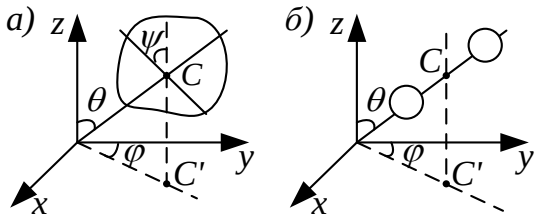


Figure 2.1 – Degrees of freedom of molecules:
a) rigid body (three Cartesian coordinates (x, y, z) and three angles (θ, φ, ψ);
b) diatomic molecule (three Cartesian coordinates (x, y, z) and two angles (θ, φ)

Values (x_c, y_c, z_c) change in the translational motion of the body and called translational degrees of freedom. Angles (θ, φ, ψ) change in the rotational motion of the body are called rotational degrees of freedom. System two rigidly interconnected material points (diatomic molecules) has five degrees of freedom (x_c, y_c, z_c, θ, φ) (see Fig. 2.1, b). When rotating around its own axis of the molecule does not change position, so you just need to two corners.

Equipartition principle states: no one type of motion of the molecule has no advantage over the other, so any degree of freedom of the molecule has in the average the same amount of energy:

$$\langle \varepsilon \rangle = \frac{1}{2} \kappa T . \qquad (2.7)$$

Then the average energy of thermal motion of molecules:

$$\langle \varepsilon \rangle = \frac{i}{2} kT, \tag{2.8}$$

where

$$i = i_{tr} + i_r + 2i_v, \tag{2.9}$$

where i_{tr} - the number of translational degrees of freedom;

i_r – the number of rotational degrees of freedom;

i_v – number of oscillatory (vibrational) degrees of freedom. In (2.9) is the sum of the number of translational, the number of rotational, and twice the number of vibrational degrees of freedom of a molecule. At low temperatures the vibrational degrees of freedom unexcited and the number of degrees of freedom equal to: $i = 3$ – for monatomic molecules $i = 5$ – for diatomic molecules, $i = 6$ – for three (or more) atomic molecules (i.e. as a solid body).

The internal energy of an ideal gas of mass m in accordance with the equipartition principle is defined by the formula

$$U = N\langle \varepsilon \rangle = N \frac{i}{2} kT = \frac{m}{\mu} \cdot \frac{i}{2} RT \cdot \tag{2.10}$$

Heat capacity of the body is called a value equal to the amount of heat needed to increase body temperature by one Kelvin. In molecular physics mostly uses the concept of molar heart capacity of gas. For gases, two types of capacity exist: at constant volume – C_V and at constant pressure – C_p. Molar heat capacity is called an amount of heat equal to the amount of one required to increase the temperature of one mole of a substance by 1 *K*:

$$C_V = \frac{i}{2} R \ , \qquad C_p = C_V + R \tag{2.11}$$

From the comparison C_V and C_p the physical meaning of constant *R* results: it equal to the work performed by one mole of a gas when it is heated by one Kelvin at isobaric process *(p = const*).

Adiabatic process – a process that takes place without heat transfer gas from the external environment (i.e., *dQ* = 0). If we put *dQ* = 0 in the equation of the first law of thermodynamics

$$dQ = \frac{m}{\mu} \cdot C_V dT + pdV, \qquad (2.12)$$

we get the equation adiabatic of ideal gas:

$$pV^{\gamma} = const, \qquad (2.13)$$

where $\gamma = {C_p}/{C_v} = {(i+2)}/{2}$, γ – adiabatic coefficient, which is determined by the number of degrees of freedom of the molecule *i.*

Proces*s* that can be done in the reverse direction so that the system has gone through the same states as the direct, but in reverse order is called reversible process. Reversible processes have the property: if the direct sense system receives heat *dQ* and performs work *dA*, then in the reverse srnse system gives off heat *dQ′ = dQ* and work done on her *dA′ =dA*. Note that reversible processes do not exist in the nature, but there are processes very close to them.

Efficiency of heat engine. Thermodynamic came first as the science of thermal machines. The task of thermodynamic – creation perfective heat engine – a device that converts thermal (internal) energy into mechanical work. Any heat engine is a system that performs a specific cycle and consists of the working substance, heater and cooler.

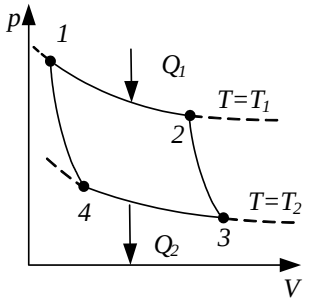


Figure 2.2 – Carnot Cycle

During the cycle the working substance expands and then shrinks. To work for the cycle was positive, with extending working substance the heat Q_1 to it is given, (determined quantitatively by area under top part of the cycle, see Fig. 2.2 and at the compression – is took the heat Q_2 (determined quantitatively area under the lower part of the cycle). The difference between the heat received and returned by the working substance is transformed into useful work. The efficiency of a heat engine

$$\eta = \frac{Q_1 - Q_2}{Q_1}. \qquad (2.14)$$

The second law of thermodynamics indicates direction thermodynamic processes. It states: impossible processes, the only end result of which would transfer heat from the body less hot to hotter bodies.

If the heat transfers from the less heated bodies to hotter is not only the end result of this process, the transition is possible, such as in the refrigerator, where heat is transferred from the less heated bodies to hotter, but the work during process is performed.

Carnot cycle. From the second law of thermodynamics implies that only reversible process which is accompanied by heat exchange with a heat reservoir at a constant temperature, is isothermal process, occurring at the temperature of reservoir.

So reversible cycle, during which the system enters the heat exchanger between two heat reservoirs, must consist of two isotherms and two adiabats. Adiabatics are steeper isotherms and close the two isotherms in the cycle. This cycle was put into consideration Carnot and he had obtained his name (see Fig. 2.2), and the heat engine that operates at Carnot cycle is called an ideal heat engine. The efficiency of the Carnot cycle is determined by the formula

$$\eta = \frac{T_1 - T_2}{T_1}, \qquad (2.15)$$

where T_1 – temperature of the heater; T_2 – temperature cooler.

We thus arrive at the statement Carnot's theorem: Efficiency of all reversible heat engines that work under identical conditions (i.e., at the same temperature of the heater and cooler) is the same.

Efficiency of a heat engine that operates by irreversible cycle is always less than the efficiency of the machine that runs on reversible cycle if the conditions of the two machines are identical. This is because the work is complete irreversible cycle of less than reversible.

Entropy. From the expression for the Carnot cycle efficiency (2.14), (2.15) follows directly

$$\frac{Q_1}{T_1} + \left(-\frac{Q_2}{T_2}\right) = 0. \qquad (2.16)$$

Monotonous function of state S, the complete differential which is defined as

$$dS = dQ/T, \qquad (2.17)$$

it is called the entropy of the system.

Thus, for a reversible process

$$\sum_{1\to 2} \Delta S = S_2 - S_1,$$

where S_1 – entropy in the initial state; S_2 – entropy in the final state.

The physical meaning of entropy follows from the statistical definition of entropy, which brought Boltzmann:

$$S = k \ln w, \qquad (2.18)$$

where k – Boltzmann constant; w – thermodynamic probability of the system that equals to the number of microscopic states that can be implemented given macroscopic state of the system. System state, which is a relatively small number of ways, called orderly. State, which is a significant number of ways, called disorderly.

Thus, entropy is a measure of chaos (disorder) in the system. The greater the entropy – the more chaos in the system.

For given the irreversible processes, formula (2.17) acquires form of inequality

$$\Delta S \geq \frac{\Delta Q}{T}, \qquad (2.19)$$

where the equality sign is taken for reversible processes and the inequality – for irreversible.

2.5 Transport phenomena in gases

In thermodynamically nonequilibrium systems, there are special irreversible processes, called transport phenomena. These phenomena include diffusion, thermal conductivity, and internal friction or viscosity.

In all these phenomena is important average mean free path of the molecules of gas. The minimum distance between the centers of two molecules when they collide is called the effective diameter of a molecule (see Fig.2.3). Effective diameter d molecules decrease with increasing temperature, because the speed of the thermal motion of gas molecules increases. Average path between two consecutive collisions molecules called mean free path λ. It is determined by the formula

$$\lambda = \frac{1}{\sqrt{2}\,\pi\, d^2 n}\,. \qquad (2.20)$$

where n – density of molecules.

Internal friction. If the velocity of ordered motion of the molecules in the gas changes from layer to layer (i.e. layers move with different speeds), the forces of friction between the layers arise (see Fig.2.4). These forces of friction are called forces of internal friction and are defined by the formula

$$F = \eta \frac{du}{dz} S\,, \qquad (2.21)$$

where η – viscosity;

$\frac{du}{dz}$ – velocity gradient orderly movement of molecules, a quantity which characterizes change in velocity in the perpendicular direction of the vector $\vec{u}$;

S – surface of layers that are confined between them.

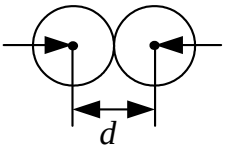


Figure 2.3 – Effective diameter of a molecule

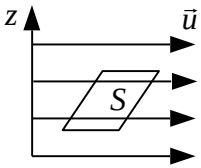


Figure 2.4 – Internal friction in gases

The viscosity is numerically equal to the force that acts on a unit area with a unit velocity gradient.

Molecules of gas pass due to thermal motion from layer to layer and carry with them a momentum. As a result, the velocity of the slow layer increases, and faster layer decreases. Transfer momentum from one layer to another causes internal friction.

In the molecular-kinetic theory derived formula for viscosity

$$\eta = \frac{1}{3}\rho < \mathrm{v} > \lambda, \qquad (2.22)$$

where ρ – density of gas; <v> – mean thermal speed of molecules; λ – mean free path of the molecules.

Thermal conductivity of gases. If along some direction the temperature of gas does not remain constant, then along this direction heat q flows in the direction of decreasing temperature (see Fig. 2.5):

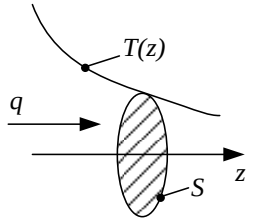


Figure 2.5 – Thermal conductivity of gases

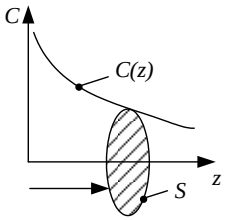


Figure 2.6 – Diffusion in gases

$$q = \chi \frac{dT}{dz} S, \qquad (2.23)$$

where q – the flow of heat, i.e. heat which is transported through surface S during unit time, $\frac{dT}{dz}$ – temperature gradient, which characterizes the rate of change of temperature along the z-axis, χ– thermal conductivity,

$$\chi = \frac{1}{3} < \mathrm{v} > l \rho c_V$$

which is equal to the heat flux through a unit area with unit temperature gradient,. c_V – specific heat capacity.

Due to the thermal (chaotic) motion, each molecule transfers from a place of higher temperature to a place with

lower temperature energy $<\varepsilon> = ikT/2$. When the temperature equalized, the flow of heat stops.

Diffusion in gases. If the concentration c (i.e., the mass per unit volume) of gas along some direction z in space (see Fig. 2.6) is changed, then due to thermal motion of molecules will occur alignment process of concentration, which is accompanied by mass transfer. As a result, in the direction of decreasing concentration occurs mass flow:

$$\mu = \frac{dm}{dt} = -D\frac{dc}{dz} \cdot S,$$

where $\frac{dm}{dt}$ – the flow of mass, i.e. the mass that is transferred through the area during unit time;

$\frac{dc}{dz}$ – concentration gradient, which characterizes the rate of change of concentration along the axis z;

D – diffusivity, which is numerically equal to the mass through a unit area with unit gradient of concentration. When the concentration is aligned, the diffusion stops.

CHAPTER 3

ELECTROSTATICS

3.1 The electric field in the vacuum

Electric charge is an inherent property of elementary particles, as well as their weight, it is a quantitative characteristic electric interaction. In nature, there are two types of charges: positive and negative.

According to the law of conservation of charge total charge electric isolated system is constant value.

Coulomb's law. Force of interaction between two point charges is determined by Coulomb's law:

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2}, \qquad (3.1)$$

$$\vec{F}_{12} = -\vec{F}_{21},$$

where r – distance between point charges;

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ (F/m) –electric constant;

q_1, q_2 – magnitudes of point charges;

F_{12} – the force acting from the first charge on the other;

F_{21} – the force acting from the second charge on the first (see Fig. 3.1).

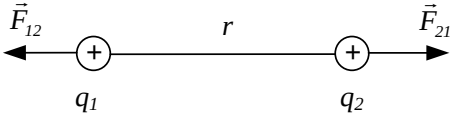


Figure 3.1 – Interaction of two point charges

When the size of charged bodies is negligible compared with the distances between them such charges are called point charges.

Electric field. Charges that are at rest interact between them through an electric field. Each charge creates in the surrounding space an electric field – a form of matter that is generated by charged bodies and provides interaction between them.

The electric field strength. The force of interaction between the charges (see Fig. 3.1) depends on the magnitude of both charges and therefore can not be characteristic of the field. The characteristic electric field at any point is the ratio of this force to the charge placed at a given point of the field

$$\vec{E} = \frac{\vec{F}}{\vec{q}}, \qquad (3.2)$$

This vector quantity $\vec{E}$ is called the electric field strength and is the force characteristics of the electric field at each point.

By direction of the intensity vector the direction of the force acting on a positive charge is received. Strength of electric field numerically equals to the force that acts on a single positive charge.

For a point charge

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}. \qquad (3.3)$$

where r – distance from the charge to the observation point.

The principle of electric field superposition. The field strength of the charge is equal to the vector-sum strengths fields generated by each of the charges separately

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_N. \qquad (3.4)$$

In the case where the charge is distributed, for example, along the length l of some linear charge density $\tau = \frac{dq}{dx}$, field strength of elementary charge $dq = \tau dx$ at point field at a distance r according to (3.3) will be:

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\tau dx}{r^2} \frac{\vec{r}}{r}.$$

The resultant field strength generated by all the charged line is defined by the vector sum of all the elementary strength are created at a given point each element of line, i.e. by the integral

$$\vec{E} = \int_0^l d\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \int_0^l \frac{\pi dx}{r^2} \frac{\vec{r}}{r} \qquad (3.5)$$

Strength lines of electric field. The electric field can be described with the aid of the strength lines of the field (of short $\vec{E}$ lines):

1) strength line start on positive and finish on the negative charges, or sent to infinity, or come from infinity;

2) vector $\vec{E}$ is directed along the tangent to the strength line at each point of the strength line ;

3) the density of lines is proportional to electric field strength.

Flux of vector strength field is determined by formula

$$\Phi_E = \int_S E_n \, ds$$

where E_n – the projection of the vector $\vec{E}$ on the direction of normal at each point of the surface S (see Fig. 3.2). The physical meaning of the vector flux consist in the next: it determinates the number of strength lines, intersecting the surface of integration S. For closed surfaces integration, as the positive normal is taken outside normal (by convexity, as shown in Fig. 3.2.)

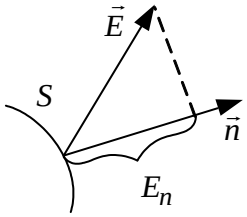


Figure 3.2 – Electric field intensity flux through a surface

Gauss`s theorem. The flux of electric field strength vector through a closed surface equals to the algebraic sum of the charges enclosed by this surface divided by the electric constant ε_0:

$$\oint_S \vec{E} d\vec{S} = \frac{\Sigma q_i}{\varepsilon_0}. \qquad (3.6)$$

Gauss theorem lets to determine the electric field charge distributed in a volume, along the surface – for bodies of regular geometric shape, for example:

1. For a field of infinite uniformly charged plane with surface charge density σ (i.e. charge per unit surface area: $\sigma = \frac{q}{S}$ (*C/m*), using the theorem, we obtain

$$E = \frac{\sigma}{2\varepsilon_0}. \qquad (3.7)$$

2. In the case of two oppositely charged surfaces outside the plates the field lines creating by plates are directed opposite to one-another so outside the plates $E = 0$. Between the plates field strength lines have the same direction, so

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

Work done by electrostatic field on a charge. Potential. Electrical force is central. It can displace the electrical charges only in direction of force lines. So elementary work done by electrostatic field created by charge q on the charge q_0 will be equal

$$dA = Fdr \qquad (3.8)$$

Work at displacement of charge q_0 when it is moved from one point to another is defined by integral sum of elementary works on all the way

$$A = \int F dr = \frac{q_1 q_0}{4\pi\varepsilon_0} \int_{r_1}^{r_2} \frac{dr}{r^2} = \frac{q_1 q_0}{4\pi\varepsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] \qquad (3.9)$$

Analyzing the expression (3.10), we can conclude: the work forces of the electric field does not depend on the shape of the trajectory, and depends only on the position of the initial 1 and final 2 points of movement, the electrostatic field is a potential.

This important conclusion can be expressed in general analytic form

$$\oint_l \vec{E} d\vec{l} = 0, \qquad (3.10)$$

which is the mathematical formulation of the circulation vector theorem: circulation of the electric field strength vector is zero. The sign $\oint$ means integration along a closed path.

As we know from mechanics, conservative force work equals decrease of potential energy, i.e.

$$A_{12} = W_{p1} - W_{p2}. \qquad (3.11)$$

Comparing expressions (3.9) and (3.11), we can conclude that the potential energy of interaction between two point charges is determined by the formula

$$W_p = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r}.$$

The ratio of potential energy to the charge is an energy characteristic of the field at this point is called potential:

$$\varphi = \frac{W_p}{q}. \qquad (3.12)$$

Potential numerically equals to the potential energy of a single positive charge at a given point of the field.

The potential of a point charge

$$\varphi = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}. \qquad (3.13)$$

The potential of the system of point charges equals the algebraic sum of potentials created at a given point field each charge separately

$$\varphi = \varphi_1 + \varphi_2 + ...\varphi_N, \quad (3.14)$$

The work done by the forces of the electric field on the charge displaced from the point 1 of potential φ1 to point of potential φ2 can now be represented as

$$A_{12} = q \cdot (\varphi_1 - \varphi_2) \quad (3.15)$$

Relation between strength field and potential. Between strength and potential of the electric field there is a relationship similar to the relationship between force and potential energy (1.36) was obtained in mechanics

$$\vec{F} = -\nabla E_P, \quad (3.16)$$

where ∇ – the gradient operator. Utilising formulas (3.2), (3.12), we obtain

$$\vec{E} = -\vec{\nabla}\varphi. \quad (3.17)$$

solve the inverse problem and according to the known intensity to determine the potential difference

$$\varphi_1 - \varphi_2 = \int_1^2 E_l dl, \quad (3.18)$$

where E_l – projection of the vector $\vec{E}$ on the direction of the vector $d\vec{l}$.

For a homogeneous electric field $\vec{E} = const$,

$$\varphi_1 - \varphi_2 = Ed, \quad (3.19)$$

where *d* – distance between points 1 and 2, which are measured along the strength line.

An imaginary surface all of whose points have the same potential is called an equipotential surface. At each point of the equipotential surface vector directed along the normal to it in the direction of decreasing potential.

3.2 Electric field in dielectrics

Dielectric are called substances that are unable to conduct electrical current because it does not have of free charges.

The polar and non-polar dielectrics. We introduce the concept of the radius vector of the centers of gravity of positive $\vec{r}^{\,+}$ and negative $\vec{r}^{\,-}$ charges of the molecule (see Fig. 3.3). Then we can assume that all the positive charge of the molecule is concentrated in the center of gravity of positive charges and negative – in the center of gravity of negative charges. In the result obtain

$$\vec{r}^{\,-} = \frac{\Sigma\, q_i^{-}\vec{r}_i^{\,-}}{-q},\ \vec{r}^{\,+} = \frac{\Sigma\, q_i^{+}\vec{r}_i^{\,+}}{q} \qquad (3.20)$$

where $\vec{r}_i^{\,+}$ - average in time vector position of *i*-th positive charge,

$\vec{r}_i^{\,-}$ - average in time vector position of *i*-th negative charge.

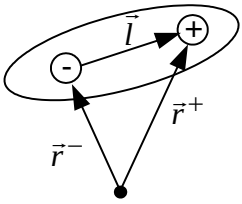


Figure 3.3 – Polar and nonpolar molecules

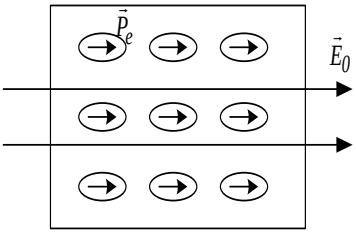


Figure 3.4 – Polarization of dielectric in an electric field

If at the absence extraneous field:

1) $\vec{r}^{\,+} = \vec{r}^{\,-}$ i.e. the centers of gravity of positive and negative charges coincide, these molecules are called nonpolar

and dielectrics formed from these molecules – nonpolar dielectrics;

2) $\vec{r}^{+} \neq \vec{r}^{-}$ i.e. the centers of gravity of positive and negative charges molecule do not coincide, these molecules are called polar and dielectrics formed from these molecules – polar dielectrics.

For polar molecules is introduced electric dipole moment of the molecule

$$\vec{p}_B = |q|\vec{l}, \qquad (3.21)$$

where q – the charge of the molecule $\vec{l}$ – vector position drawn from the center of gravity of the negative charges in the center of gravity of the positive charges of the molecule.

Molecule in an external electric field. In the external electric field:

a) non-polar molecule stretches under the action of the field and gets electric dipole moment directed along the external field proportional in value to the field strength:

b) polar molecules unfold and set its dipole moment in the direction of the vector strength of the external electric field (i.e. polar molecule behaves as a rigid dipole).

In the absence of an external electric field, the total dipole moment of a dielectric is zero: for a non-polar dielectric, the dipole moments of the molecules are zero, while for a polar dielectric, the dipole moments of the molecules are randomly oriented, resulting in no net dipole moment.

Under the influence of an external electric field dielectric polarizes – resulting dipole moment of the dielectric is different from zero: nonpolar molecules are stretched and oriented their dipole moments oriented along with the external electric field (see Fig. 3.4). Quantitative characteristic of dielectric polarization is polarizability $\vec{P}$ - electric dipole moment of the volume unity of dielectric:

$$\vec{P} = \frac{\sum \vec{p}_{ei}}{V}, \qquad (3.22)$$

where $\Sigma \vec{p}_{ei}$ - the sum of electric dipole moments in the volume *V*. Experiments revealed that polarization isotropic dielectrics is proportional to the total field strength. For isotropic dielectrics

$$\vec{P} = \chi \varepsilon_0 \vec{E}, \qquad (3.23)$$

where χ – dielectric susceptibility. $\vec{E}$ – field strength in dielectric.

Bound charges. On the surface of the insulator due to polarization a bound charges appear (see Fig. 3.5).

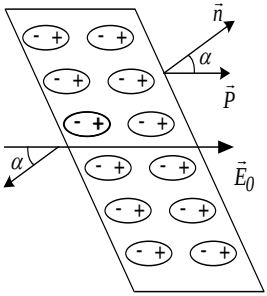


Figure 3.5 – Polarization charges on the surface of a dielectric

The surface density of bound charges is determined by the formula

$$\sigma' = P\cos\alpha = P_n, \qquad (3.24)$$

where P_n – polarization projection on the external normal to the surface of the dielectric. In view of (3.23) formula (3.24) takes the form

$$\sigma' = \chi \varepsilon_0 E_n, \qquad (3.25)$$

where E_n – the projection of the field strength inside the dielectric in the vicinity of the surface of the outer normal to the surface. A formula (3.25) determines not only the value of σ, but also its sign.

The field in the dielectric. According to the principle of superposition electric field strength inside the dielectric consists of two components:

$$\vec{E} = \vec{E}_0 + \vec{E}', \tag{3.26}$$

where $\vec{E}_0$ – the strength of the external field, $\vec{E}'$ – field created by bound charges $E' = \sigma'/\varepsilon_0$, which were appeared on the dielectric surface, with its polarization.

Vector $\vec{E}'$ is always directed opposite to the vector $\vec{E}_0$, so in the scalar form:

$$E = E_0 - E' = E - \frac{\sigma'}{\varepsilon_0} < E_0, \tag{3.27}$$

i.e. dielectrics always weaken the electric field. From (3.26) and (3.27) with (3.25)

$$\varepsilon_0 E_0 = \varepsilon_0 \vec{E} + \chi \varepsilon_0 \vec{E} = (1+\chi)\varepsilon_0 \vec{E} = \varepsilon \varepsilon_0 \vec{E} \tag{3.28}$$

where $\varepsilon = 1 + \chi$ is the relative dielectric constant, which shows how many times the electric field inside the dielectric smaller of external field: $\varepsilon = E_0/E$; $\varepsilon\varepsilon_0\vec{E} = \vec{D}$

where $\vec{D}$ – the vector displacement.

From the expression (3.28) implies that the vector displacement is the same in vacuum and in the substance, because it is determined only by free charges.

Pyro-, piezo-, ferroelectrics. In some crystals the total dipole moment different from zero even in the absence of external electric field. About such dielectrics say they are spontaneous polarized. There are some dielectrics in which spontaneous polarization depends on the temperature. At heating or cooling of dielectrics on its surfaces appear electric

charges opposite sign, which can be detected. This phenomenon is called pyroelectricity. The most famous is the crystal tourmaline. Piezoelectric effect is the appearance of electric charges on the surfaces of the crystal by applying a mechanical stress. This is so-called direct effect. There is a reverse effect, which is that under the influence of electric field applied to the simple crystal mechanical deformation appears. Ferroelectrics are a subset of pyroelectrics, exhibiting spontaneous polarization only within a specific temperature range. Above a characteristic temperature, known as the Curie temperature, this spontaneous polarization vanishes.

3.3 Conductors in an external electric field

Conditions of equilibrium of charges on the conductor. Conductors differ from the dielectrics by presence into them of a large number of free electrons. These electrons can move freely inside a metal under the influence of the electric field, but they can not leave the conductor. In the case of action, the electrostatic field on the conductor electrons move until the moment when they will distribute along its surface so that inside of the conductor electric field becomes zero. So, one of the conditions of equilibrium of the charge on the conductor is the vanishing of the electric field inside the conductor:

$$E = 0. \qquad (3.29)$$

From equation (3.17) and condition (3.29) follows that $\nabla\varphi = 0$ i.e. within the conductor $\varphi = const.$ It means that the whole volume of conductor including the surface is equipotential. Then, based on the conditions of orthogonality equipotential lines and strength lines can be confirm that for the field on the surface of the conductor will be different from zero only normal component

$$E = E_n, \qquad (3.30)$$

which is the second condition of equilibrium charges.

Thus field inside the conductor is absent, but on the surface directed along the normal to the surface.

Distribution charges into the conductor. A charge, imparted to a conductor distributes itself over its surface according to two equilibrium conditions of charges.

At equilibrium charges in any place inside the conductor can not be surplus charges. They distribute on the outer surface of the conductor (see Fig. 3.6). Charge density increases with increasing positive surface curvature (convexity) and decreases with increasing negative curvature (concavity).

Conductor in an external electric field. Place uncharged conductor in an electric field. The charge carriers in the conductor will start to move. At the ends of the conductor charges of the opposite signs accumulate. They are called *induced charges* (see Fig. 3.7). Redistribution of charges ends

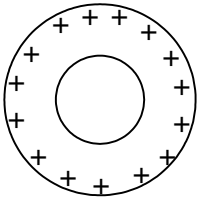


Figure 3.6 – Charge distribution on the surface of a conductor

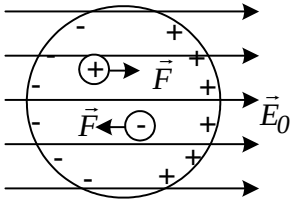


Figure 3.7– Charge redistribution in a conductor in an electric field

when both conditions equilibrium of charge on the conductor are performed.

Induced charges distribute on the outer surface conductor and the field inside the conductor is zero, so that the field induced charges compensate the external field inside the conductor. This statement remains true for hollow conductor. Electrostatic protection is based on that, i.e. the protection of certain places of space from external electric fields.

Capacitance conductor. Physic quantity, characterized by the ability of the conductor to accumulate electrical charges called capacitance:

$$C = \frac{q}{\varphi}, \tag{3.31}$$

where q – the charge on the conductor; φ – potential of the conductor, i.e. potential electric field at a point on the surface of the conductor. From the equilibrium charges conditions on the conductor it follows that the potentials of all points on the surface of the conductor are the same.

At the same potential of the conductor the more electric charge may be accumulated on the conductor. Capacitance of conductor depends on the shape, size of the conductor and electric properties of the medium surrounding the conductor (relative dielectric permittivity). For example, for a spherical conductor

$$C = 4\pi\varepsilon\varepsilon_0 R, \tag{3.32}$$

where R – radius of the sphere; ε – relative dielectric constant of the medium.

Capacitors. Isolated conductors have low capacitance. Even for ball of Earth size capacitance is about 700 μF. For storing charges are capacitors – devices which with little sizes accumulate large in magnitude charges. Capacitor structure is based on the fact that conductor capacitance increases when there are some conductors brought to it. Conductors that form a capacitor (plates) should be formed to concentrate electric-field between them. Then approach of external bodies does not effect on the capacitance of the capacitor. Possible only three such forms of plates, three types of capacitors: plane – parallel, cylindrical and spherical. Capacitance capacitor is determined by the formula

$$C = \frac{|q|}{U}, \tag{3.33}$$

where q – the charge of one of the plates, U – potential difference between plates. For a plane capacitor

$$C = \frac{\varepsilon\varepsilon_0 S}{d}, \qquad (3.34)$$

where S – square of plate; d – distance between the plates; ε – relative permeability of the medium between the plates.

Capacitance of capacitor battery:

a) At parallel connection (Fig. 3.8);

b) At series connection (Fig. 3.9).

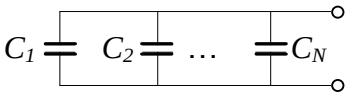


Figure 3.8 – Capacitance at parallel connection

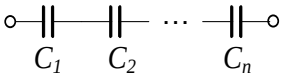


Figure 3.9 – Capacitance at series connection

$$C = \sum_{i=1}^{N} C_i \,. \qquad \frac{1}{C} = \sum_{i=1}^{N} \frac{1}{C_i} \,. \qquad (3.35)$$

Energy of the charged conductor

$$W = \frac{1}{2} q\varphi = \frac{q^2}{2C} = \frac{C\varphi^2}{2}, \qquad (3.36)$$

where q – charge of the conductor; φ – potential of conductor, C – capacitance. Energy charged conductor accumulates in electrical field around the conductor.

The energy of a charged capacitor:

$$W = \frac{1}{2} qU = \frac{q^2}{2C} = \frac{CU^2}{2}, \qquad (3.37)$$

where q – the charge on one of the plates; U – potential difference between the plates, C – capacitance of capacitor. Energy carrier charged capacitor is the electric field between the capacitor plates.

Electric field energy. The density of electric field energy in a dielectric, i.e. energy per unit volume is determined by formula

$$w = \frac{W}{V} = \frac{\varepsilon \varepsilon_0 E^2}{d} \qquad (\text{J/m}^3), \qquad (3.38)$$

where ε – relative dielectric constant of the medium, E – electric field. The density of electric field energy is proportional to the square of the electric field strength.

PART 2

ELECTRODYNAMICS. WAVES. OPTICS. QUANTUM MECHANICS

CHAPTER 4

ELECTRODYNAMICS

4.1 Steady electric current

Orderly movement *t* of electric charges is called electric current. By direction of the current direction of movement of the positive charges is taken.

Current strength. The average current strength during Δt

$$I_{cep} = \frac{\Delta q}{\Delta t}, \tag{4.1}$$

where Δq – a charge that passes through a cross section of the conductor during Δt.

Instantaneous current strength:

$$I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}. \tag{4.2}$$

Current density. At the uneven distribution of charges along the cross section of the conductor passing through it, the concept of current density introduces:

$$j = \frac{dI}{dS}, \tag{4.3}$$

where *dS* – elementary area perpendicular to the direction of current. Knowing the current density j, we can to determine current strength

$$I = \int_S j_n dS, \tag{4.4}$$

where $j_n = j \cos\alpha$ – projection of current density on the normal to the surface *S* at each point of the surface.

Electromotive force (e.m.f.). The forces of the electric field can to move a positive charge from points with more potential to points with less potential.

In a closed circuit, along with lots of less potential must to be a lots with growth potential. In areas with an increase of the potential movement of positive charges is possible by means only of a non-electric forces (called *extraneous forces*).

Force of any nature that may to move positive charges to the direction of growth potential, belongs to a class of extraneous forces.

Extraneous forces can be characterizing by the work done by these forces on a single positive charge. This value is called electromotive force and it equals

$$E = \frac{A}{q}, \tag{4.5}$$

Extraneous forces act in the power and make to move positive charges from the lesser potential (from a minus sign) to larger (to a plus).

Ohm's law for a homogeneous circuit section. Current strength flowing in the circuit section is directly proportional to the voltage drop in the section and inversely proportional to the electrical resistance of the circuit section.

$$I = \frac{U}{R}. \tag{4.6}$$

For conductors, electrical resistance is determined by the formula

$$R = \rho \frac{l}{S},$$

where ρ – resistivity of the conductor; l – length of the conductor; S – cross-sectional area of the conductor.

Ohm's law for the an nonuniform circuit section. Voltage. Consider the circuit section, which contain e.m.f. (Fig. 4.1). Total work A_{12} displacement of charge along the section circuit consists from work of extraneous forces in the power source A_{12}^{e} and work of electric forces of current external of source A_{12}^{E}

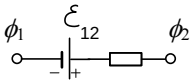


Figure 4.1 – Non-uniform section of a circuit

$$A_{12} = A_{12}^{e} + A_{12}^{E}. \qquad (4.7)$$

Dividing the obtained formula by the charge q which moves on the section, and introducing the following values: $U_{12} = {}^{A_{12}}/_{q}$ – *voltage* on the section of circuit, $\varphi_1 - \varphi_2 = {}^{A_{12}^{e}}/_{q}$ – potential difference and $E_{12} = {}^{A_{12}^{e}}/_{q}$ – e.m.f. of the circuit, rewrite the formula (4.7) as

$$U_{12} = (\varphi_1 + \varphi_2) + E_{12}. \qquad (4.8)$$

Thus, the voltage on the section of circuit is equal to the algebraic sum of potential difference and e.m.f.. If the direction of current source coincides with the direction of the current, then $E_{12} > 0$, and if it does not coincide, then $E_{12} < 0$. According to (4.6) we can rewrite relation (4.8)in the form $IR_{12}=(\varphi_1-\varphi_2) + E_{12}$, or

$$I = \frac{\varphi_1 - \varphi_2 + E_{12}}{R + r} \qquad (4.9)$$

where R_{12} is the sum of external resistance R and internal resistance of e.m.f. r., i.e. $R_{12} = R + r$.

Obtained formula is Ohm's law for an nonuniform circuit section containing e.m.f .Let us join the points with potentials φ_1 and φ_2, then we obtain closed circuit. Potential difference becomes equal zero and from (4.9) we find $I = \frac{E_{12}}{R + r}$.

Obtained formula is Ohm's law for the closed circuit containing e.m.f.

Ohm's law (4.6) can be written in differential form:

$$\vec{j} = \sigma \vec{E},$$

where $\vec{j}$ – current density vector, density $\vec{E}$ – electric field strength in the conductor; $\sigma = 1/\rho$ – *conductivity* of the conductor.

The resistivity of most metallic conductors ρ depends on the temperature according to the Law

$$\rho = \rho_0(1+\alpha t), \tag{4.10}$$

where ρ_0 – resistivity at the temperature t = 0°C, α – temperature coefficient of resistance. At room temperature, ρ varies in direct proportion to the absolute temperature T = t °C + 273.

**Joule`s law.** When current flows along the wire last heats. This heat called Joule`s heat, is determined by the Joule`s law. For direct current

$$Q = I^2 R\Delta t,$$

where I – direct current strength, R – resistance of the conductor;

$\Delta t = t_2 - t_1$ – time during which current flows.

For current, alternating in time

$$Q = \int_{t_1}^{t_2} I^2 R dt, \tag{4.11}$$

Kirchhoff`s Rules. Multiloop electrical circuits can be easily calculated with the two rules of Kirchhoff.

The first rule. Algebraic sum of the currents in the junction equals zero

$$\sum I_k = 0.$$

Junction – a branch point of the circuit, where more than two conductors. Currents coming into the junction, are taken with a plus sign, and those that go out – with a negative sign.

Second rule. Algebraic sum of voltage drops in the loop (the products the currents by resistances) is equaled to algebraic sum of e.m.f. acting in this loop:

$$\sum I_k R_k = \sum E_k.$$

Electric current in gas (gas discharge) is possible only if there are carriers – free electrons, positive or as negative ions. These charged particles are formed by ionization – under action of ionizers – high temperature, ultraviolet radiation, X-ray, nuclear radiation, etc.

Along with the process of ionization in the gas, the reverse process – recombination (the union charged particles in neutral atoms or molecules) takes place. Usually the intensity of recombination is proportional to the intensity of ionization: the higher ions and free electrons appears per unit time, the more of them formed neutral atoms and molecules.

Semi-self-maintained discharge is formed external ionizer and occurs only in the presence of their actions. At low voltages between the cathode and the anode flowing current is weak and discharge called silent. In this case, the number of pairs of ions of opposite sign, appearing for unit of time equals to the number of pairs that recombine at the same time. The current depends on the speed of ions, which is proportional to the field strength. When the voltage achieves a certain threshold magnitude, current strength reaches saturation. The magnitude of saturation current is determined by the power of the ionizer and does not change as long as an electric field starts to accelerate ions and electrons to speeds that can ionize the gas. It leads to an increase of current, at which silent discharge transfers in self – maintained discharge.

Self – maintained discharge begins when a voltage commit U_z (voltage ignition or breakdown). Gas begins to shine, and the discharge occurs in the absence of an initial ionizer as glow, arc, spark, high-frequency, or corona discharges.

Glow discharge occurs at low pressure ~ 10 Pa and has the form shown in Fig. 4.2.

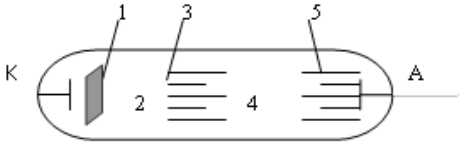


Figure 4.2 – Discharge tube

Main areas of discharge: 1 – cathode film, 2 – cathode dark space, 3 – glow luminous column, 4 – Faraday dark space, 5 – positive pole luminous ionized gas. In areas 1 – 4 is a sharp voltage drop. In the cathode dark space greatly accelerated electrons and positive ions, which beat out from the cathode secondary electrons.

4.2 Magnetic field in a vacuum

Electric currents interact between them. An electric current creates a magnetic field in the surrounding space, which acts on another conductor with a current, made in this field.

Magnetic field – is a form of matter that is generated by moving charges and acts on moving charges.

Magnetic induction is the main characteristic of the magnetic field. To measure the magnetic field is used the circuit current small size (test current loop). Circuit with a current is characterized by a magnetic moment $\vec{p}_m$, which is determined by the formula

$$\vec{p}_m = I\,S\,\vec{n}\,, \qquad (4.12)$$

where S – area unclosed by the loop, $\vec{n}$ – unit positive normal, orientation in space which is associated with the direction of the current in the circuit by rule of right screw (see Fig. 4.3).

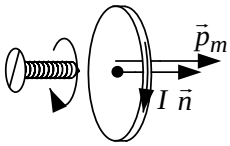


Figure 4.3 – Orientation in space of the positive unit normal $\vec{n}$ to a flat current loop I

When making a circuit with a current in a magnetic field on it exerts a torque, under which the loop turns and takes in equilibrium position. The ratio of the maximum rotational moment of the forces acting on the contour with the current at the magnetic moment of the loop is the force characteristics of the magnetic field at the point where is founded test loop, called magnetic induction

$$\vec{B} = \frac{M_{\max}}{p_m} \vec{n}_{equil}$$

Direction vector is defined by the equilibrium position the positive normal to the contour. The unit of measurement of magnetic induction in the SI is 1 Tesla (T).

The flux of the magnetic induction. The magnetic field is customary represented by lines of magnetic induction. The magnetic induction flux Φ of the vector induction $\vec{B}$ is determined by the number of lines of magnetic induction cutting surface S

$$\Phi = \int_S B_n ds, \qquad (4.13)$$

where B_n – the projection of the magnetic induction on the normal in every point of the integration surface S. The unit of measurement of magnetic flux in the SI system is 1 Weber.

For a homogeneous magnetic field (vector $\vec{B}$ at all points of field equal in magnitude and direction) flux of the vector $\vec{B}$ across a flat surface area S is defined by the formula

$$\Phi = B\,S\cos\alpha = B_n S,$$

where α – the angle between the magnetic induction vector and the normal to the plane.

Gauss`s theorem for magnetic induction vector states that magnetic induction flux through a closed surface S equals zero

$$\Phi = \oint_S B_n ds = 0. \qquad (4.14)$$

This result is a consequence of the fact that the lines of magnetic induction are always closed. Thus, any closed surface is crossed by line twice: once in one direction, the second in opposite one and the resulting induction flux is zero.

Law of Bio-Savart-Laplace determines the magnetic induction of current element $\vec{dl}$ i.e. small section of conductor length dl, the direction of which coincides with the direction of current:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}}{r^3}, \qquad (4.15)$$

where $\vec{r}$ – vector position which joints the current element with the point of observation, μ_0 = 4π·10-7 *H/m* – magnetic constant (see Fig. 4.4).

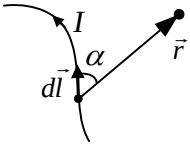


Figure 4.4 – Observing point radius

In scalar form:

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\alpha}{r^2}$$

where α – the angle between the vectors $\vec{dl}$ and $\vec{r}$.

The field of straight wire with current. For the magnetic field the superposition principle holds

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \ldots \vec{B}_N,$$

i.e. induction vector of currents equals the vector sum of the fields, that each conductor with current creates separately.

Breaking conductor to the elements current and using Bio-Savart-Laplace`s law and the superposition principle, we can determine the magnetic induction of any conductor with current. For direct infinitely long conductor as a result we obtain the formula for induction

$$B = \frac{\mu_0 I}{2\pi r},$$

where r – shortest distance between the current and the observation point.

The magnetic induction lines of the line current are a system of concentric circles surrounding the wire (see Fig. 4.5).

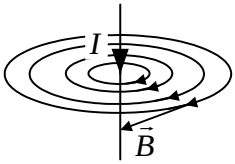


Figure 4.5 – Magnetic field lines of a straight current-carrying wire

For limited by the length of straight conductor induction is defined by formula

$$B = \frac{\mu_0 I}{4\pi r}(\cos\alpha_1 + \cos\alpha_2), \qquad (4.16)$$

where α_1 and α_2 – angles between the conductor and the vectors position $\vec{r}_1$ and $\vec{r}_2$, respectively, drawing from observation point to the ends of the conductor (see Fig. 4.6).

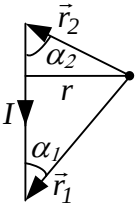


Figure 4.6 – Limited in length by a straight conductor

When $\alpha_1 \to 0$ and $\alpha_2 \to \infty$ (i.e. at the transition to an unlimited wire) formula (4.16) turns into the formula for infinitely long wire.

The circulation of the magnetic induction – is integral along a closed path L:

$$\oint_L \boldsymbol{B}_L \boldsymbol{dl} = \boldsymbol{\mu}_0 \sum_{k=1}^{N} \boldsymbol{I}_k$$

where B_L – projection of vector $\vec{B}$ on the elementary area of integration $d\vec{l}$, i.e. the direction of the tangent at each point of the contour (see Fig. 4.7).

If the path of integration L uncloses several currents (see Fig. 4.8), the circulation of the magnetic induction equals the product of magnetic constant μ_0 on the algebraic sum of currents that are unclosed by the contour of integration:

$$\oint_L B_l dl = \mu_0 \sum_{k=1}^{N} I_k \,. \qquad (4.17)$$

In the formula (4.17) the currents, which direction is associated with the direction of integration by right screw rule are positive.

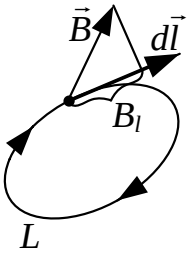


Figure 4.7 – Contour for calculating the circulation of vector $\vec{B}$

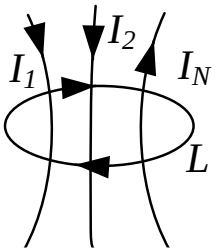


Figure 4.8 – The integration contour L encloses several currents

Field of a solenoid. Solenoid coil is called a coil which the diameter is much smaller than its length. Using the formula (4.18) for the circulation of the magnetic induction, it is easy to obtain the formula for the magnetic field inside an infinitely long solenoid

$$B = \mu_0 n I, \qquad (4.18)$$

where n – number of turns per unit length of the solenoid. Thus, the magnetic field of an infinitely long solenoid is uniform and fully concentrated inside the solenoid.

For a limited solenoid magnetic field line are closed through free space. Inside the solenoid magnetic field is almost homogeneous and well described by the formula (4.18). Homogeneity of field broken only near the ends of the solenoid.

4.3 Interaction between currents and magnetic field

Ampere force – a force acting on an element of current $I\vec{dl}$ in magnetic field

$$d\vec{F}_A = I\, d\vec{l} \times \vec{B}. \qquad (4.19)$$

Or in scalar form $dF_A = I\, B\, dl \sin\alpha$, where α – the angle between the vectors $\vec{B}$ and $d\vec{l}$ (see Fig. 4.9).

For a conductor with current, located in a uniform field $\vec{B}$ =const.

$$F_A = I\, B\, l\, \sin\alpha.$$

Lorentz`s force – the force acting on a moving charged particle in a magnetic field. It is determined by the formula

$$F = qvB,$$

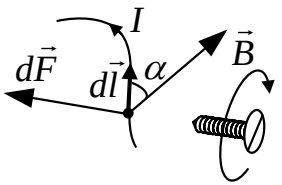


Figure 4.9 – The direction of the Ampere force

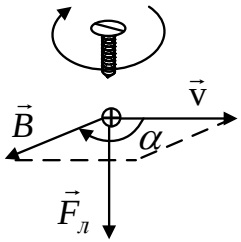


Figure 4.10 – The direction of the Lorentz force

or in scalar form $F = qvB\sin\alpha$, where α – the angle between the vectors $\vec{B}$ and $\vec{v}$ (see Fig. 4.10).

Since $\vec{F} \perp \vec{B}$ Lorentz`s force does not do work and can not change the magnitude of the particle velocity. It changes only the direction of the velocity vector.

Current loop in a magnetic field. If you bring in a magnetic field current loop, then from the field to the loop operates torque (see Fig. 4.11)

$$\vec{M} = \vec{p}_m \times \vec{B}.$$

Under the action of the torque $\vec{M}$ loop turns and orients its magnetic moment $\vec{p}_m$ along the magnetic induction $\vec{B}$.

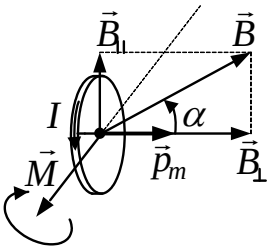


Figure 4.11 – Torque acting on a current loop from an external magnetic field

4.4 Magnetic field in matter

Magnetization of the substance. Any substance is a magnetic that is capable under the influence of a magnetic field to acquire a magnetic moment (magnetized). Magnetized substance produces its own magnetic field with induction $\vec{B}'$ which is superimposed on the magnetic field $\vec{B}_0$ produced by currents (macroscopic currents). As a result, the magnetic fields in matter

$$\vec{B} = \vec{B}' + \vec{B}_0 .$$

Magnetization of the magnetic is characterized by magnetization – $\vec{J}$ magnet moment per unit volume

$$\vec{J} = \frac{\sum_{\Delta V} \vec{p}_{m_i}}{\Delta V},$$

where $\vec{p}_{m_i}$ - the magnetic moment of the individual molecule or atom that are within the volume ΔV.

Ampere`s hypothesis. To explain the magnetization of bodies Ampere suggested that in the molecules or the atoms circulate currents creating magnetic moments. Due to the chaotic orientation of the magnetic moment of individual molecules, the magnetic moment of the body in the absence of external magnetic field is equaled zero.

Under the influence of an external magnetic field with the induction $\vec{B}_0$ the magnetic moments of the molecules or atoms are set along the external magnetic field. The substance becomes magnetized, i.e. its summing magnetic moment of the molecules becomes different from zero. Molecular currents magnetized substances set up its own magnetic field, with induction $\vec{B}'$.

Description field of the magnetics. The circulation of the vector in the matter written in the form

$$\int_L B_l dl = \mu_0 (\sum I)_L + \mu_0 (\sum I_M)_L , \qquad (4.20)$$

where $\left(\sum I\right)_L$ – the sum of known macroscopic currents flowing in conductors enclosed by loop of integration L and forming the external magnetic field with induction $\vec{B}_0$;

$\left(\sum I_M\right)_L$ – the sum unknown microscopic currents of molecules unclosed by the contour of integration L and forming a magnetic field of magnetized substance.

At the formula (4.20) to determine the magnetic field in a substance, we must know the molecular currents, and to determine the molecular currents, it is need to know the induction field in the substance. There is a vicious circle. This indicates that with aid only one magnitude can not to describe a field in matter. It must enter additional magnitude that is associated with the induction, but is determined only through known macroscopic currents.

The sum of the molecules currents in the right side of (4.20) can be expressed with help of vector $\vec{J}$. Then the expression (4.20) can be rewritten as

$$\int_L (\frac{\vec{B}}{\mu_0} - \vec{J})d\vec{l} = (\sum I)_L .$$

Magnitude $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{J}$, is called the vector magnetic field, for which the circulation is

$$\oint_L H_l dl = \sum I , \qquad (4.21)$$

i.e. is determined only by known macroscopic currents in conductors.

In the vacuum magnetization. $\vec{J} = 0; \vec{B} = \mu_0 \vec{H}$.

In substance, as experience shows, for all substances $\vec{J} = \chi \vec{H}$, where χ - the magnetic susceptibility of the substance.

Then in the matter $\vec{B} = \mu_0 (1 + \chi) H$.

Introducing the relative magnetic permeability of material $\mu = 1 + \chi$, we obtain the relationship between induction $\vec{B}$ and intensity $\vec{H}$ in the matter $\vec{B} = \mu \mu_0 \vec{H}$.

The relative permeability

$$\mu = \frac{B}{B_0} \qquad (4.22)$$

shows how many times the magnetic induction $\vec{B}$ in the material is greater than the induction $\vec{B}_0$ in a vacuum.

Types of magnetics. Depending on the relative permeability μ all substances are divided into three groups:

1) $\mu = const < 1$ - diamagnetic;
2) $\mu = const > 1$ - paramagnetic;
3) $\mu >> 1$ - ferromagnetic.

Diamagnetic substances are substances whose atoms do not have magnetic moments. Under the action of external magnetic field there is a rotation of electronic orbits around the vector of magnetic conduction $\vec{B}_0$. That results the origin of the induction magnetic moments of atoms, directed against the external field $\vec{B}_0$. The magnetic field of substance $\vec{B}'$ directed opposite $\vec{B}_0$ appears. Resulting field in a substance $B = B_0 - B' < B_0$.

Paramagnetic. If the magnetic moment of atoms different from zero, a substance called paramagnetic. The external magnetic field is trying to establish the magnetic moments of atoms along the vector $\vec{B}_0$. A positive magnetic moment of the substance greater than the negative induction moment appears. The resulting magnetic moment of substances is positive , that is directed along the external field B_0. Resulting field in matter $B=B_0+B'>B_0$

Ferromagnets are substances that can be magnetized even in the absence of an external magnetic field (iron, nickel, cobalt, etc.). Magnetization *J* of ferromagnetic depends on the strength of the magnetic field *H* in the matter (see Fig. 4.12). As the magnetic field strength *H* increases, the magnetization *J* of a ferromagnetic material increases until it reaches a constant value, at which point the ferromagnetic material achieves saturation.

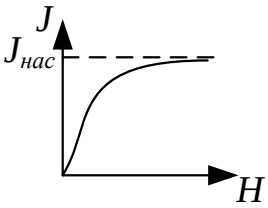


Figure 4.12 – The dependence of the magnetization J of ferromagnets on the magnetic field strength H

In the nonlinear dependence of *J* from ferromagnets observed hysteresis phenomenon. If we bring up magnetization to saturation (point 1 in Fig.4.13) and then diminish the magnetic field strength, the induction will no longer follow the initial curve 0-1, but will change according to curve 1-2. As a result, when the strength of the external field vanishes (point 2), the magnetization does not vanish and is characterized by the quantity B_r called *residual induction.* When an alternating magnetic field acts on ferromagnetic, the magnetic induction changes in accordance with curve *1-2-3-4-5-1* (Fig.4.13) called *hysteresis loop.*

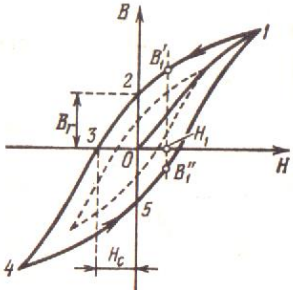


Figure 4.13 – Hysteresis loop

Nature ferromagnetism lies in its domain structure. At certain conditions in the crystal that compel the magnetic moments of electrons align parallel to each other. As a result, a

regions of spontaneous magnetization arise. Dimensions domains ~ $10^{-4} \div 10^{-3}$ *cm.*

When a ferromagnetic material is removed from a magnetic field, partial orientation of the domains is stored. As a result, residual induction is observed. When a ferromagnet is heated at a certain peculiar temperature T_C, for a given ferromagnetic, due to the thermal motion of atoms, the domain structure is destroyed and the ferromagnetic becomes a paramagnetic. The temperature at which this occurs is called the Curie point. For iron $T_C = 768℃$ for nickel $T_C = 365℃$.

4.5 Electromagnetic induction

The law of electromagnetic induction. Lenz`s Rule. An e.m.f. induction and an induced electric current are produced in a closed conducting loop when the flux of magnetic induction through the surface enclosed by this loop changes. This phenomenon is called electromagnetic induction phenomenon.

The value of induced current, and hence the e.m.f. induction does not depend on the way by which a magnetic flux changes, and is determined only by the speed of its change

$$\xi_i = -\frac{d\Phi}{dt}. \qquad (4.23)$$

Formula (4.23) is called the law of electromagnetic induction.

Direction of induced current is determined by Lenz's rule: the induced current is always directed so that its magnetic flux resists changing of magnetic flux that caused it.

With increasing of the magnetic flux through the loop field lines induced current directed towards the magnetic induction of the external field, with decreasing magnetic flux – to the same side. If the circuit consists of N turns, which are connected in series, the e.m.f. induction is:

$$\xi_i = \frac{d\psi}{dt}$$

where $\psi = \sum_{i=1}^{N} \Phi_i$ – total magnetic flux, which is the sum of magnetic fluxes through all the turns of the loop.. If all turns are the same then $\psi = N\Phi$, where Φ – magnetic flux that penetrates one turn.

Induction currents excited in massive solid conductor by alternative magnetic fields, called eddy current. If the resistance of massive conductors is small, eddy currents can reach significant values. To reduce eddy currents and prevent energy loss in heating transformer cores due to eddy currents, the cores are assembled with thin insulation sheets.

Inductance. Magnetic flux generated by circuit current is proportional to the current strength that flows in the loop: $\psi = L\,I$ **.**

The coefficient of proportionality L is called the *inductance* of a loop. Inductance describes the ability of loop to form a magnetic flux. Inductance depends on the shape, size of the loop and magnetic properties of the medium surrounding the loop. Unit of inductance is 1 Henry (H) it is inductance of the loop, which at a current of 1 Ampere forms on magnetic flux 1 Weber (Wb).

Inductance of the solenoid is given by formula

$$L = \mu\,\mu_0\,n^2 V$$

where $n = N / l$ – number of turns per unit length of a solenoid; V – volume of the solenoid.

Self-induction. Electric current flowing in the loop forms a magnetic flux that permeates this loop. When you change the current, the magnetic flux will change and the magnetic flux and, consequently, e.m.f. induction. will occur in the circuit. This phenomenon is called self-induction. E.m.f. of self-induction in the absence of a ferromagnetic is determined by the formula

$$\xi_L = -L\frac{dI}{dt}.$$

Direction of current of a self-induction, as induction current is determined by the Lenz`s rule. By increasing the current in the circuit self-induction current goes to meet him, and with a decrease – in the same direction.

The energy of a magnetic field. The magnetic field as a form of matter and is a carrier of energy. The energy of a magnetic field is determined by the formula

$$W_m = \frac{LI^2}{2}.$$

Energy density $w = W/V$, i.e. energy of the magnetic field per unit volume, is defined by the formula

$$w_m = \frac{\mu\mu_0 H^2}{2} = \frac{HB}{2}. \qquad (4.24)$$

CHAPTER 5

OSCILLATIONS AND WAVES

Processes that are periodically repeated in time, and in which the physical quantity characterizing the oscillatory process again deviates from zero and returns to it again, are called oscillatory.

In nature, there are mechanical, electromagnetic, electromechanical and other oscillations.

In an oscillatory process, the coordinate, angular deviation from the equilibrium position, electric charge on the capacitor plates, and so on, change.

5.1 Free undamped vibrations

Free mechanical oscillation. Mechanical oscillation system called the system, on which quasielastic (restoring) force acts

$$F = -kx, \tag{5.1}$$

where x – the displacement from equilibrium of the body;
k - coefficient of proportionality.

The minus sign in the formula (5.1) means that the *restoring force* is always directed towards the equilibrium position. For spring pendulum, the restoring force is the force of elasticity, and the coefficient *k* is the coefficient of elasticity of the spring. Since the system has one force (if we neglect by force of friction), according to Newton's second law: $ma = -kx$, so by definition

$$a = \frac{dV}{dt} = \frac{d^2x}{dt^2}, \tag{5.2}$$

and the equation of motion (5.2) can be reduced to the form

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0, \tag{5.3}$$

where $\omega_0 = \sqrt{k/m}$ - natural frequency oscillatory system – spring pendulum. Accordingly, the period of free oscillations

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}.$$

The solution of the homogeneous linear differential equation of the second order (5.3) is harmonic functions (sine or cosine):

where *x* -instantaneous displacement;

a- amplitude (maximum value of displacement);

$\omega t + \alpha$ – phase of oscillations;

α – the initial phase;

$\omega = 2\pi\nu$ - cyclic frequency of oscillations;

T – the period of oscillations.

In oscillatory system, which is derived from the equilibrium in the absence of friction forces free mechanical harmonic oscillations with constant amplitude occur.

Natural (free) electromagnetic oscillations occur in the oscillatory circuit without active electrical resistance when we charge capacitor and disconnect it from the source of voltage. According to the second Kirchhoff`s rule voltage drops across the capacitor equals e.m.f. of self-induction, which occurs in coil of induction

$$U_c = -L\frac{dI}{dt}. \qquad (5.4)$$

By definition, $I = dq/dt$,

$$U_c = q/C \ . \qquad (5.5)$$

Substituting (5.5) into equation (5.4), we obtain

$$\frac{d^2q}{dt^2} + \omega_0^2 q = 0 \ . \qquad (5.6)$$

Thus, for a charge on the capacitor we have

$$q = q_m \cos(\omega_0 t + \alpha), \qquad (5.7)$$

where the *natural frequency* of oscillation $\omega_0 = 1/\sqrt{LC}$,

L – inductance of coil,

C – capacitance of capacitor.

Period of free electromagnetic oscillations: $T_0 = 2\pi\sqrt{LC}$

Oscillations of voltage on the capacitor and the current in the circuit at free oscillations is described by formulas

$$u_c = U_m \cos(\omega_0 t + \alpha),$$

$$i = I_m \cos(\omega_0 t + \alpha + \pi/2),$$

i.e. current leads the voltage across the capacitor in phase by $\pi/2$.

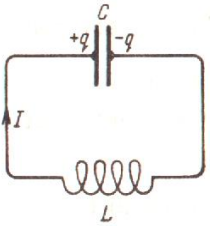


Figure 5.1 – LC oscillatory circuit (lossless)

5.2 Damped oscillations

Damped mechanical oscillations. In any real mechanical oscillation system there are forces of resistance

$$F_r = -r\mathrm{v},$$

where v – velocity of the body, r – the resistance coefficient.

Take into account the friction force from Newton's second law we obtain

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0.$$

In this case, the displacement of the body from the equilibrium position is described by the formula

$$x = a_0 e^{-\beta t}\cos(\omega t + \alpha)$$

where a_0 – amplitude at the initial time;

$\beta = r/2m$ - damping factor;

$\omega = \sqrt{\omega_0^2 - \beta^2}$ - frequency of damped oscillations;

$a_0 e^{-\beta t}$ – amplitude at the any moment of time.

Accumulated in oscillatory system energy is spent on work against the friction forces, energy of the system decreases and oscillations are damped (see Fig. 5.2).

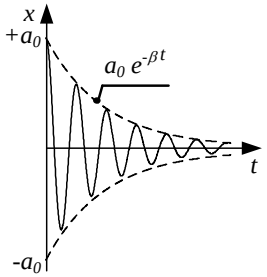


Figure 5.2 – Graph of damped oscillations

Thus, the presence of the resistance in oscillatory system not only leads to the damping of oscillations, but reduced their frequency. Damped oscillations exist in the oscillatory system at the condition $\beta < \omega_0$. When $\beta > \omega_0$ resistance forces in oscillatory system are so large that oscillations in the system do not arise.

Damped electromagnetic oscillations occur in the oscillatory circuit with ohmic resistance *R*. Energy stored in the circuit during the charging capacitor decreases because of the heating resistance. As a result, oscillations in the circuit are damped.

For a circuit that consists of elements connecting in series, in accordance with the second rule of Kirchhoff, we obtain the equation

$$U_c + IR = -L\frac{dI}{dt}, \qquad (5.8)$$

that after minor transformations takes form.

$$\frac{d^2q}{dt^2} + 2\beta\frac{dq}{dt} + \omega_0^2 q = 0, \qquad (5.9)$$

where $\beta = R/2L$ – demption factor.

Changing of the charge on the capacitor is described by the formula

$$q = q_0 e^{-\beta t}\cos(\omega t + \alpha), \qquad (5.10)$$

which is a solution of equation (5.9). Cyclic frequency damped electromagnetic oscillations are given by the expression

$$\omega = \sqrt{\omega_0^2 - \beta^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}.$$

When $\beta > \omega_0$ the capacitor is aperiodically discharged because circuit resistance is too large and oscillations in the circuit are impossible.

5.3 Forced oscillations

Forced mechanical oscillation. These oscillations occur in the oscillatory system under action of external periodic force:

$$F = F_0 \cos\omega t.$$

In this case, after some time in an oscillatory system are installed harmonic oscillations

$$x = a\cos(\omega t - \varphi)$$

with the frequency of the external periodic force ω. The amplitude of oscillation is given by formula

$$a = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}}.$$

At a certain frequency $\omega_{рез}$, called the resonance frequency forced oscillation amplitude reaches the maximum value. This phenomenon is called *resonance*. Graph of the amplitude of the forced oscillations of frequency is called resonance curve (Fig.5.3).

The resonance frequency forced oscillation is determined by the formula

$$\omega = \sqrt{\omega_0^2 - 2\beta^2}\ .$$

If $2\beta^2 > \omega_0^2$ resonance in the system is not observed. It must to marked that resonance in the mechanical system is harmful. Cases are known when bridges collapsed owing to the marching of columns of soldiers over them.

Forced electrical oscillations. To produce forced oscillations of a system, elements *R, L, C* connected in series (see Fig.5.4), an external alternated voltage must be exerted on it $U=Um\ cos\omega t.$

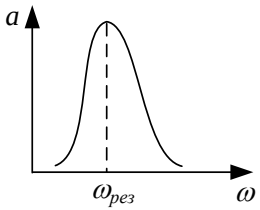


Figure 5.3 – Resonance curve or amplitude-frequency characteristic of a resonant system

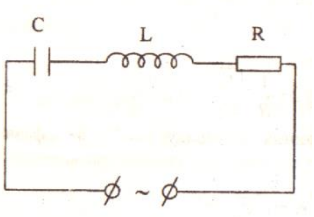


Figure 5.4 – Series RLC circuit with ac current (or ac voltage source**)**

in accordance with the second rule of Kirchhoff we obtain equation.

$$\frac{d^2q}{dt^2} + 2\beta\frac{dq}{dt} + \omega_0^2 q = Um\ cos\omega t.$$

A partial solution of this equation has the form

$$q = q_m \cos(\omega_0 t - \psi),$$

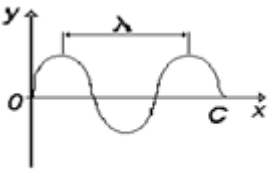


Figure 5.5 – Wavelength (graphical representation)

Current changes according law

$$I = -\omega q_m \sin(\omega_0 t - \psi) = I_m \cos(\omega_0 t - \varphi),$$

where $I_m = -\omega q_m$,

$$\varphi = \psi - \pi/2,$$

$$I_m = \frac{U_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}},$$

$$\tan\varphi = \frac{\omega L - 1/\omega C}{R}.$$

The amplitude of the current I_m has a maximum at $\omega L = 1/\omega C$, consequently, the resonance frequency for the current coincides with natural frequency ω_0.

5.4 Elastic waves

The process of propagation oscillations in an elastic medium is called the elastic wave. The particles of the medium in which the wave propagates are not transferred by the wave. They only oscillate about their equilibrium positions.

If the particles of medium oscillate in a plane that is perpendicular to the direction of wave propagation, these waves are called *transverse*. If the direction of oscillation of

particles coincides with the direction of propagation – it is a *longitudinal* wave.

Transverse waves occur only in a medium that is able to resist shear deformation. Therefore, in liquids and gases only longitudinal waves can appear, in solids r both: longitudinal and transverse waves may appear.

Locus of the points, reached by the wave process at the moment of time *t*, called the *wavefront.* Locus of the points that oscillate in the same phase, called the *wave surface*. In dependence of the shape of the wave surface the waves divide into *flat, cylindrical, spherical*. Plane waves are formed by a flat source of oscillations, cylindrical by a linear and spherical by a point source.

Equation of a plane wave. Particular attention in the theory is paid to the study of harmonic waves – infinite sinusoidal waves. Such waves do not exist, but having studied the properties of simple waves, we can analyze more complex wave processes that occur in nature. The value of such an idealization at the study of the waves can be compared with the notion of a material point, which plays a fundamental role in the mechanics.

Let us establish the relationship between the displacement of particles in an elastic medium that are found in oscillatory motion, and their distance from the vibration source, located in the plane *Y0Z* (Fig. 5.5). We assume that the oscillations of the source (plane *Y0Z*) is harmonic.

$$y = A\sin \omega t, \tag{5.11}$$

where A – amplitude of wave, ω – oscillation frequency. Each point of the medium is involved in the oscillatory motion at a time when the wave reaches that point. To the point, which is at a distance x from the source, the wave comes during a time interval $\tau = \frac{x}{v}$ (v – velocity of the wave). Thus, the equation

of particle oscillations *C* (in the medium out of attenuation) can be written as

$$y = A\sin\omega(t-\tau) = A\sin\omega(t-\frac{x}{v}). \qquad (5.12)$$

Equation (5.12) allows the determination of the displacement *y* from equilibrium at any point in the medium involved in a wave process at any moment in time $t \geq \frac{x}{v}$ and is called the equation of a plane monochromatic wave. The value of the displacement *y* from position of equilibrium is determinate by amplitude *A* and phase is determined by

$$\phi = \omega(t-\frac{x}{v}). \qquad (5.13)$$

Let us fix a complete phase value (5.13):

$$\omega(t-\frac{x}{v}) = const, \qquad (5.14)$$

and determine the speed of its movement. Find the derivative of (5.14)

$$\frac{dx}{dt} = v. \qquad (5.15)$$

The left side of equation (5.15) is the speed of movement of certain values of the phase. Thus for a monochromatic wave propagation velocity of wave *v* coincides with the speed of movement phase. In this regard, it is called phase velocity. Taking into account, that $\omega = 2\pi\nu = 2\pi/T$, $\nu = \lambda/T$ rewrite equation (5.12) in the form

$$y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right). \qquad (5.16)$$

Equation (5.16) shows a double periodicity of the wave process: over time – with period T and in the space with period

λ. In wave theory is introduced the concept of wave number $k = \frac{2\pi}{\lambda}$, then from (5.16) we obtain

$$y(x,t) = a\cos(\omega t - kx). \qquad (5.17)$$

Standing waves. If the elastic medium are several sources of oscillation, in the region of crossing waves oscillations of points of a medium which are caused by each of the waves superpose onto one another without disturbing one another. Result of addition (the resultant wave) depends on the ratio between the phases, periods and amplitudes of the superposed waves. A very important is the case of the addition of two (or more) waves having constant difference phases. Such waves and sources that emit them are called coherent. Addition of coherent waves is called interference. The very important case of interference is observed when waves are reflected from obstacles. The phenomenon of interference is often encountered in acoustics, radio. Standing waves occur in moving objects, and in which there are various sources of vibration. Let us suppose, that in the direction of the X axis, in a medium where decrement of amplitude of a plane wave is absent, a plane wave propagates

$$y_1 = A\sin\left(\omega t - 2\pi\frac{x}{\lambda}\right)$$

and meets on his way flat boundary separating media. After her reflection occurs reflected wave that propagates in the opposite direction

$$y_2 = A\sin\left(\omega t + 2\pi\frac{x}{\lambda}\right).$$

The resulting displacement is the sum of both components y_1 and y_2

$$y = y_1 + y_2 = 2A\cos\frac{2\pi x}{\lambda}\sin\omega t\ . \qquad (5.18)$$

The factor in front of $\sin\omega t$ in the formula (5.18) shows that at every point of a standing wave harmonic oscillations have the same frequency ω. Magnitude $2A\cos\frac{2\pi x}{\lambda}$ plays the role of the amplitude which depends on the position of point (x- coordinate). At those points where $\cos\frac{2\pi x}{\lambda}=0$ the amplitude of the oscillations equals zero. In these points

$$x=(2k+1)\frac{\lambda}{4}, \tag{5.19}$$

where $k=1,\ 2,\ 3...$, these points are called the nodes of the standing wave are situated. At those points, where $\left|\cos\frac{2\pi x}{\lambda}\right|=1$ amplitude has a maximum equaled $2A$ antinodes are situated. For antinodes:

$$x=k\frac{\lambda}{2},\ (k=1,\ 2,\ 3...). \tag{5.20}$$

The distance between adjacent antinodes, as seen from the formula (5.20)

is $\frac{\lambda}{2}$. The distance between adjacent nodes is also equaled $\frac{\lambda}{2}$. It should be noted that the phase in (5.18) does not depend on the coordinates. This means, that unlike the propagating wave, where at any time adjacent points of a medium are different phases. in the standing wave all points between neighboring nodes oscillate with the same phase. Wave equation. Equation of a plane wave (5.17) is a solution of the differential equation of second order in time in partial derivatives, which is called the wave equation:

$$\frac{\partial^2 x}{\partial z^2}=\frac{1}{\mathrm{v}^2}\frac{\partial^2 x}{\partial t^2}. \tag{5.21}$$

Electromagnetic waves. Vortex electric field. Analyzing the phenomenon of electromagnetic induction Maxwell concluded that the cause of appearance of induction current in the loop is the vortex electric field setting up by change of the magnetic flux. Strength lines of the electric field is locked, i.e. field is vortex. If $\vec{E}$ the intensity of the vortex electric field, the e.m.f. of induction ξ in a closed loop L according to the circulation theorem is

$$\xi = \oint_L \vec{E} d\vec{l} \ .$$

Then the law of electromagnetic induction $\xi = -\frac{d\Phi}{dt}$ can be written in the integral form:

$$\oint_L \vec{E} d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} d\vec{S} \ . \qquad (5.22)$$

Vector $\vec{B}$ in (5.22) depends on two variables, time and coordinates, so we have to the right symbol partial time derivative.

Displacement current. Maxwell generalized theorem of circulation of vector $\vec{H}$ for stationary currents in the case of variable electromagnetic fields, Suggesting that the alternating electric field creates the alternating magnetic field, Maxwell introduced the concept of displacement current density $\vec{j}_d$

$$\vec{j}_d = \frac{\partial \vec{\mathrm{D}}}{\partial \mathrm{t}}, \qquad (5.23)$$

We underline the fact that the term “displacement current” is purely conventional. In essence, displacement current is a time-varying electric field. The only reason for calling the quantity given by Eq. (5.23) a ‘current’ is that the dimension of this quantity coincides with that of current density.

As a result, the concept of total current density as the sum of the densities of the conduction current $\vec{j}_c$ and the displacement current $\vec{j}_d$ is introduced

$$\vec{j} = \vec{j}_c + \frac{\partial \vec{\mathrm{D}}}{\partial \mathrm{t}}.$$

Using the concept of displacement current, it is possible to explain how electric current flows between the plates of a capacitor where there are not of the charges carriers. Displacement current ensures passing alternating current through a capacitor.

In a result of this generalization the theorem of circulation vector *H* (4.21), acquires the form:

$$\oint_L \vec{H} d\vec{l} = \int_S \left(\vec{j} + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}.$$

For the space between the plates conduction current equals zero, so

$$\oint_L \vec{H} d\vec{l} = \int_S \left(\frac{\partial \vec{D}}{\partial t} \right) d\vec{S}. \qquad (5.24)$$

Electromagnetic field. Electromagnetic wave. From equation (5.22) implies that the change in time of the magnetic field causes a change in time electric field. From equation (5.24) implies, that the change in time of the electric field causes the appearance of an alternating magnetic field in time. The consequence of equations (5.22), (5.24) is the conclusion, that the alternating electric and magnetic fields are indissolubly linked and create electromagnetic field. Originating, electromagnetic field begins to propagates in space. The process of propagation of electromagnetic field in space is called an electromagnetic wave.

So in the electromagnetic wave, vectors' strength of the electric field and the magnetic field vary. Vectors oscillate in

phase, i.e., simultaneously reach maximal values and simultaneously reach zero (see fig.5.6)

$$\vec{E} = \vec{e}_y E_m \cos(\omega t - kz + \alpha),$$

$$\vec{H} = \vec{e}_x H_m \cos(\omega t - kz + \alpha), \tag{5.25}$$

where $\vec{e}_x, \vec{e}_y$ – orts along the coordinate axes

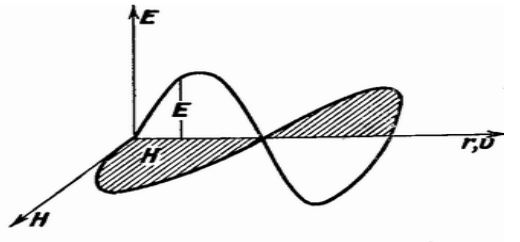


Figure 5.6 – Plane electromagnetic wave

Vectors $\vec{E}$ and $\vec{H}$ are perpendicular to each other and vary in the plane, which is perpendicular to the direction of wave propagation, i.e. electromagnetic waves belong to the class of transverse waves, and the vectors $\vec{E}$ and $\vec{H}$ form a right-handed system (see Fig. 5.6). Calculating the second partial derivatives in time and co-ordinates of x and (5.25) we obtain the wave equations

$$\frac{\partial^2 E_y}{\partial z^2} = \varepsilon\, \varepsilon_0\, \mu\, \mu_0 \frac{\partial^2 E_y}{\partial t^2},$$

$$\frac{\partial^2 H_x}{\partial z^2} = \varepsilon\, \varepsilon_0\, \mu\, \mu_0 \frac{\partial^2 H_x}{\partial t^2}. \tag{5.26}$$

Comparing equations (5.21) and (5.26) we can conclude that the speed of propagation of electromagnetic waves in a medium $v = \dfrac{1}{\sqrt{\varepsilon\varepsilon_0\mu\mu_0}}$, in vacuum $v = \dfrac{1}{\sqrt{\varepsilon_0\mu_0}} = c$,

where $c = 3 \cdot 10^8\ {}^{m}/_{s}$ – the speed of light in vacuum.

Thus the vacuum speed of electromagnetic waves coincides with the speed of light. Hence, Maxwell concluded that light waves are electromagnetic waves.

So, in a medium with permittivity and permeability ε, μ

$$v = \frac{c}{\sqrt{\varepsilon\mu}} = \frac{c}{n}, \qquad (5.27)$$

where *n*-absolute refractive index of medium.

The amplitudes of the vectors $\vec{E}$ and $\vec{H}$ are related by the expression

$$\sqrt{\varepsilon\varepsilon_0}E_m = \sqrt{\mu\mu_0}H_m.$$

The principal difference electromagnetic waves from mechanical is that mechanical waves can propagate only elastic medium, electromagnetic waves – is a type of field that can propagate in vacuum.

Dipole radiation. Let us consider the oscillating system formed by a point charge +*q* and a point charge – *q*. The dipole electric moment of this system varies in time according to the law

$$\vec{P} = \vec{P}_0 \cos\omega t,$$

where $\vec{P}_0 = q\vec{l}$, and $\vec{l}$ - arm of dipole. Together with the change of the field strength of dipole $\vec{E}$ the alternating magnetic field and the electromagnetic field are formed, which breaks away from the dipole, and at a considerable distance from the dipole has the form of a plane harmonic wave. Calculations show that intensity wave $I \sim \omega^4$. In direction of dipole axis dipole does not radiate at all, and in direction perpendicular to this axis the intensity is maximal and does not depend on the azimuthal angle.

The Doppler effect for electromagnetic waves. At the relative motion of the electromagnetic wave source and the receiver frequency of the received signal differs from radiated.

Applying the Lorentz transformation for the wave radiated by the source and received by a receiver we can obtain an expression for the angle frequency ω wave recorded by receiver that moves with velocity v relative to the source

$$\varpi = \frac{\varpi_0 \sqrt{1-(\upsilon/c)^2}}{\left[1+(\upsilon/c)^2 \cos\theta\right]}, \qquad (5.28)$$

where ϖ_0 – angle frequency of wave emitted by the source;

θ – the angle between the vector velocity V and the direction of observation (measured in the reference frame associated with the observer). In dependence on the magnitude of the angle on distinguish longitudinal and transverse Doppler effect. The first one takes place during source motion and receiver along straight line. Then θ = 0 if the receiver is removed, and $\theta = 180^0$ if it approach to the source. When θ =π/2 transverse effect is observed.

When $\upsilon \ll c$, eq. (5.28) for the longitudinal effect can be writing approximately as follows

$$\varpi = \varpi_0 (1 \pm V^2/c^2), \qquad (5.29)$$

where the plus sign corresponds to rapprochement, while the frequency of the receiver increases and negative – to the case when the distance between a source and the receiver increases the receiver frequency decreases.

For the transverse Doppler effect when the vector of the relative velocity is directed at right angle to the straight line passing through the receiver and source, the expression (5.29) takes the form

$$\varpi = \varpi_0 (1 - V^2/c^2).$$

Both, longitudinal and transverse effects observed in astrophysics in the study of moving celestial bodies. In terrestrial conditions in longitudinal Doppler effect is used in satellite geodesy for determining the change in the distance and speed of moving objects

CHAPTER 6

WAVE OPTICS

Wave optics studies optical phenomena described by the wave theory. This are phenomena of interference, diffraction, polarization and dispersion.

6.1 Light wave

The nature of the light wave is the electromagnetic wave with wavelengths in the range $\lambda = (0{,}40 \div 0{,}76) \cdot 10^{-6}$ m or in frequency range $\nu = (0{,}39 \div 0{,}75) \cdot 10^{15}$ Hz.

Photofiziological, photochemical, and photoelectric effects of light are due to the electrical components of an electromagnetic wave, so-called in optics light vector

$$\vec{E} = \vec{E}_m \cos(\omega t - kz + \alpha). \qquad (6.1)$$

Formula (6.1) describes a light wave that propagates along the z axis.

6.2 Interference of light

The phenomenon of interference. Consider the superposition of two light waves of the same direction and the same frequency

$$E_1 = E_{m1} \cos(\omega t + \alpha_1), \text{ and. } E_2 = E_{m2} \cos(\omega t + \alpha_2),$$

where α_1, α_2 - the initial-phase oscillations.

The amplitude of the resulting oscillation is determined by the expression

$$E_m^{\,2} = E_{m1}^{\,2} + E_{m2}^{\,2} + 2E_{m1}E_{m2} \cos(\alpha_2 - \alpha_1),$$

or in the intensities

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\alpha_2 - \alpha_1).$$

If the phase difference of two light waves with time does not change, these waves are called coherent.

For coherent waves at points where

1) $\cos(\alpha_2 - \alpha_1) > 0$ – resulting intensity $I > I_1 + I_2$, i.e. the resulting intensity when replaying the two waves is greater than the sum of the intensities of the individual waves;

2) $\cos(\alpha_2 - \alpha_1) < 0$ - the resulting intensity $I < I_1 + I_2$.

Thus, in case of two coherent light waves redistribution of flux in space takes place, which leads to appearance a maximum and minimum of the light intensity, alternating in space. This phenomenon is called *interference*, and alternation of maximums and minimums of intensity – the *interference pattern.* When applied two plane coherent waves the interference pattern is alternating light and dark bands.

Coherent and incoherent addition of oscillation. Intensity maximums occur at points in space where $\cos(\alpha_2 - \alpha_1) = 1$. The intensity of the light in the maximums

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2. \qquad (6.2)$$

If the intensity of the waves are the same $(I_1 = I_2)$,

$$I_{max} = 4I_1 \ . \qquad (6.3)$$

Intensity minimums appear at the points of space where $\cos(\alpha_2 - \alpha_1) = -1$. Light intensity at the minimum

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2. \qquad (6.4)$$

Under identical intensities waves

$$I_{\min} = 0. \qquad (6.5)$$

Formulas (6.2) – (6.5) define the so-called law of coherent addition of waves. For incoherent light waves the phase difference $(\alpha_2 - \alpha_1)$ arbitrarily changes in time, average value $\cos(\alpha_2 - \alpha_1) = 0$ and the resulting intensity at all points

$$I=I_1+I_2, \quad (6.6)$$

and at the same waves intensities

$$I = 2I \quad (6.7)$$

Formulas (6.6) and (6.7) give the law of incoherent addition of intensities of the waves.

Conditions for interference maxima and minima. The law of independence of light rays states that rays do not interfere with each other when they intersect. To observe interference, coherent light sources are required. In nature, such sources are rare, as most natural light sources are incoherent. Two coherent light waves can be produced by splitting a single light wave into two parts.

Let the first wave propagates in a medium of absolute refractive index n_1 and passes the point overlap geometrical path S_1, and the second one- in a medium of absolute index n_2 travels the geometric path S_2 (Fig. 6.1). *Optical path of the light beam is called the product of the geometric path to absolute refractive index*. We introduce the optical beam path $L = nS$, then

$$L_1 = n_1 S_1 L_2 = n_2 S_2$$

and the optical path difference of rays

$$\Delta L = L_2 - L_1 = n_2 S_2 - n_1 S_1.$$

Then the phase difference at the point of overlapping $\delta = {2\pi(\Delta L)}/{\lambda_0}$. Maximum of intensity are observed at points of space for which $\Delta L = \pm m\lambda_0, m = 0,1,2, \ldots$. That is the optical path difference of rays equals integer number of wavelengths.

Light intensity of minimums are observed at the points of space for which $\Delta L = \pm\left(m + \frac{1}{2}\right)\lambda_0, m = 0,1,2, \ldots$.

That is, the optical path difference of rays equals to half-integer of wavelength

Experiment of Jung. In this experiment, the light falls on the screen with two parallel slits, and the interference pattern is observed on the complete screen, located behind the first (see Fig. 6.2). The interference pattern is an alternation of light and dark bands and observed on site AB, where overlapping coherent waves (so-called field interference). On the axis of symmetry (coordinate $x = 0$) is observed light band (interference maximum).

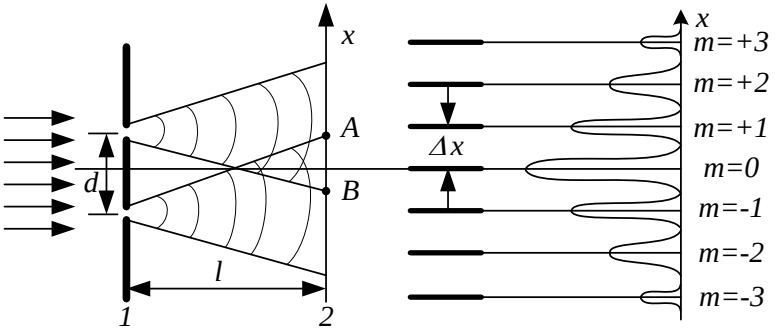


Figure 6.1 – Young's double-slit experiment

Coordinates of the maximum and minimum intensities are determined by formulas

$$x_{max} = \pm m\frac{l}{d}\lambda,\ x_{\min} = \pm(m+\frac{1}{2})\frac{l}{d}\lambda,$$

where m= 0,1,2 ... – order of interference maximum (or minimum):

l – distance from the slits to the screen:

d – distance between the slits.

When $l \gg d$ the distance between two neighboring maximums (or minimums) Δx is determinate by the formula

$$\Delta x = \frac{l}{d}\lambda. \qquad (6.8)$$

Measuring, l and d, from (6.8) is easy to determine the wavelength of light.

Interferometer. Interference phenomena is the base of operation of the interferometer. Figure 6.2 is a schematic view of a Michelson interferometer. The light beam generated by laser 1, falls on the semitransparent plate 2. Half of the incident light flux is reflected by plate 2 and falls on the mirror 3 (so – called reference beam). Second half (so- measuring beam) pass throw the plate and falls on the mirror 4. Both beams are reflected from mirrors reference – from real mirror 3, and measuring from movable mirror 4 and again fall on the plate 2. Both beams interfere according to equation (6.2) and directed to the detector 5. If the mirror 4 moves slowly 4, the detector will register maximums and minimums of intensity depending on whether the phase or antiphase are reference and measuring beams.

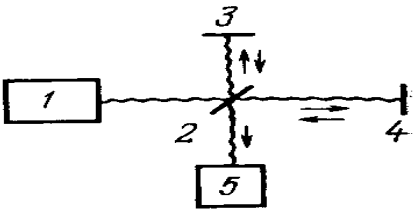


Figure 6.2 – Diagram of a Michelson interferometer

Displacement of the mirror 4 in $\lambda/2$ will correspond to the change of optical path difference between beams in $\lambda/2$. Registering the number of peaks and minimums of light at moving mirror 4, you can determine the change of the distance in quarters wavelength.

6.3 Diffraction of light

By diffraction is meant light waves bending around obstacles, when wave deviates from the straight propagation and penetrates into the region of a geometrical shadow. The

emergence of diffraction can be explained with the help of Huygens's principle: each point of the wave front is source of secondary spherical wave summarizing of which gives the position the wave front in the next moment of time (Fig. 6.3).

Rays of light propagate along the normal to the wavefront. Therefore, the diffraction of light leads to the penetration of light in the region of geometric shadow and the formation of the diffraction pattern – light and dark areas, which alternate (see Fig. 6.3).

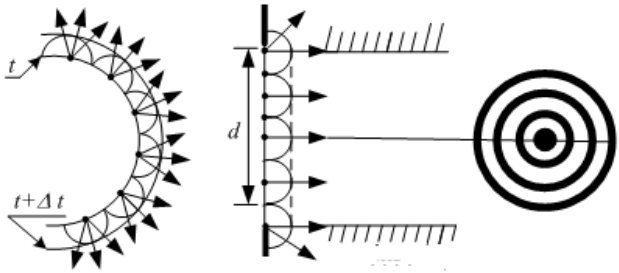


Figure 6.3 – Diffraction of light

Diffraction largely depends on the ratio of the dimensions of the obstacles d in the way of light and wavelength λ. Diffraction most more pronounced when $d \approx \lambda$..

Fresnel zone. According to Fresnel, let us divide the wave surface shown in the figure 6.4 into annular zones 1,2.3… constructed so that the optical path from the edges of each zone to point P differ by $\lambda/2$. These sites of the wave surface are called Fresnel zones. Then resulting ondulation at the observation point P by secondary waves from two adjacent zones Fresnel are in anti-phase, that differ in phase by π.

Therefore, the amplitude of the resulting oscillations generated at the observation point P by secondary waves that come from the entire wave surface, defined by the formula

$$A_p = A_1 - A_2 + A_3 - A_4 + \ldots, \qquad (6.9)$$

where A_m – amplitude formed by secondary waves that come from the m-th Fresnel zone.

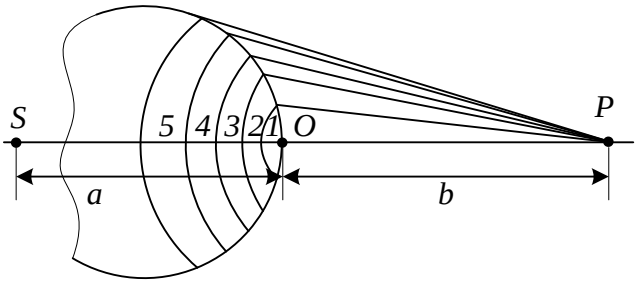


Figure 6.4 – Fresnel Zones

For point light source the wave surface is spherical and Fresnel zones for it are annular regions (except the first) with a radius of the outer boundary

$$r_m = \sqrt{\frac{ab}{a+b} m\lambda}\,, \qquad (6.10)$$

where a – the radius of the spherical surface (see Fig. 6.4);

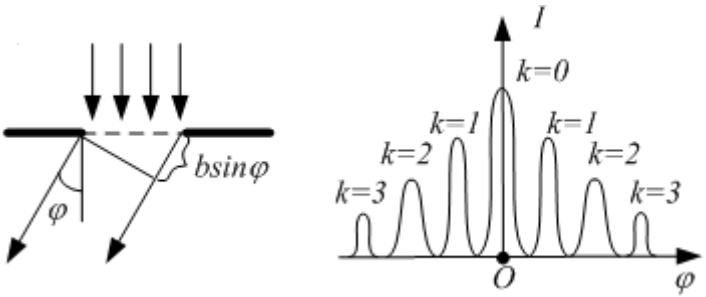
b – the distance from the top of a spherical segment (point A) to the observation point P (measured along the line connecting the source S and the point of observation P).

Fraunhofer diffraction on the slit. At the diffraction of parallel beam on the slit in the screen the rays are scattered in all the directions. Rays that go in one direction, interfering with each other and in the result on the screen is observed diffraction pattern. At normal incidence of the beam on the slit of width *b*, the difference in optical path of the extreme rays, which are the same angle diffraction φ (the angle between the normal and the plane of the slit) is $b\sin\varphi$ (Fig. 6.5, a).

At the condition

$$b\sin\varphi = \pm k\lambda\,, \qquad k=1,2,3,\ldots, \qquad (6.11)$$

the wave surface of slit is divided into 2*k* (even) number of Fresnel zones. Action of each pair zones compensate on each other and at the given diffraction angle observed diffraction minimum.



a b

Figure 6.5 – a) The path of extreme rays under normal incidence on a slit of width *b*; b) the dependence of the intensity of diffracted light on the diffraction angle φ

At the condition

$$b\sin\varphi = \pm(k+\tfrac{1}{2})\lambda, k=1,2,3,\ldots, \qquad (6.12)$$

on the slit 2*k* +1 (odd) number of Fresnel zones packs up. Action of one zone remains noncompansated and at this diffraction angle a diffraction peaks is observed.

In Fig. 6.5, b the dependence of the intensity of the diffracted light from the diffraction angle φ is shown. At the angle φ = 0 all secondary rays are in the same phase and in this point the main diffraction peak (*k* = 0) is observed. Light strips of the diffraction pattern correspond to the maximum intensity, and dark – minimum intensity.

Diffraction grating. Periodic sequence of slits in nontransparent screen forms a diffraction grating. We further assume that slits are much narrower than nontransparent plots. Oscillations that propagate from different slits are coherent and interfere. The result of superposition depends on the phase difference for two adjacent slots $d\sin\varphi$, where d - period of a diffraction grating, φ – diffraction angle (Fig.6.7). At the condition

$$d\sin\varphi = \pm m\lambda \quad (\text{m} = 0,1,2\ldots) \qquad (6.13)$$

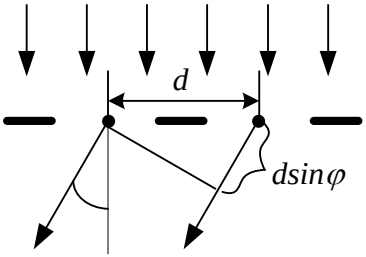


Figure 6.6 – Diffraction from a diffraction grating

For diffraction angles φ main diffraction peaks are observed. Formula (6.13) is called the *equation of a diffraction grating.*

The number (m) of principal maxima observed is determined by condition $m \le \frac{d}{\lambda}$.

Because the diffraction angle can not exceed $\frac{\pi}{2}$, the amount of maxima equal to *2m +1*.

The positions of the principal maxima depend on the wavelength. Therefore, at the fall on the diffraction grating of white light all the principal maxima of diffraction pattern, except for the central one. Will expand into a spectrum, whose violet end faces the center of the diffraction pattern, and whose red end faces outward. The amplitude of the main maximum of the grating $A = N \cdot A_i$, where A_i – amplitude at maximum for a single slit, N - the number of slits of the diffraction grating. Then the intensity of the principal maximum of the diffraction grating $I = N^2 I_i$. Thus, the diffraction grating is a spectral instrument for measuring wave length of light.

6.4 Polarization

Natural and polarized light. In natural light, vector E oscillates rapidly and chaotically in various directions in a

plane perpendicular to the direction of light propagation (Fig. 6.7, a).

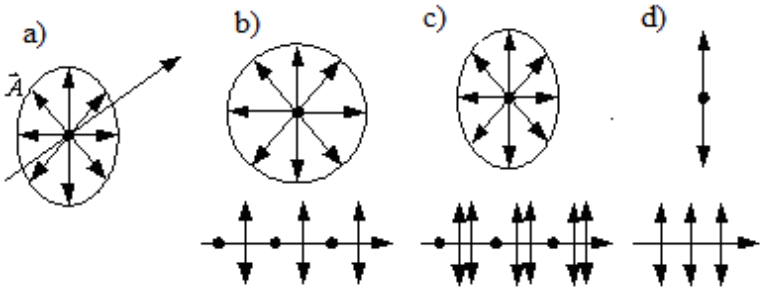


Figure 6.7 – a) Transverse oscillations of the light vector in a plane perpendicular to the ray; b) unpolarized light; c) ellipse; d) polarized light

Such light is called unpolarized. If you connect the ends of the light vectors in different directions, we'll get a circle (see Fig. 6.7, b).

Light, in which the oscillations of the light vector ordered in some way called polarized. Distinguish between partial and full polarization is so. In partly polarized light amplitude of oscillations in some direction is larger than that in the perpendicular direction, there is a partly ordering of oscillations of the light vector. If we connect the tails of the light vectors of different directions we will receive ellipse (see Fig. 6.7, c). In perfect polarized light oscillations of the light vector occur in the same direction (plane-polarized light (see Fig. 6.7, d)). Polarization of the light ray can be represented using double-headed arrows and points. Arrows indicate the oscillations of the light vector in the plane of the figure and the points – oscillations perpendicular to the plane of the drawing. For unpolarized light, density arrows and points on the rays are the same; for partly polarized light, they are different (see Fig. 6.7). The expression

$$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

is known as the degree of polarization.

For plane- polarized light $I_{min} = 0$ and $P = 1$. For natural light, $I_{\max} = I_{\min}$ and $P = 0$, for partly polarized light $0 < P < 1$

Polarizer. Natural unpolarized light is converted into polarized by using polarizers. Polarizer is a device that pass the oscillations parallel to some plane called plane of polarizer and does not completely pass the oscillations perpendicular to it (see Fig. 6.8, a). Assume that plane-polarized light of intensity I_0 falls on the polarizer (see Fig. 6.8, b)

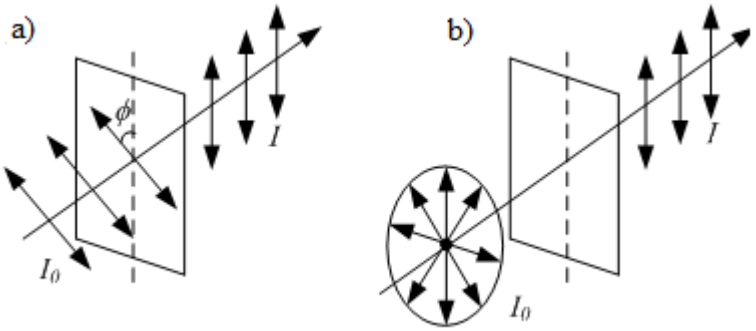


Figure 6.8 – a) Polarized light is incident on a polarizer;
b) unpolarized light is incident on a polarizer

The component of the oscillation having the amplitude $A = A_0 \cos\varphi$, where φ is the angle between the plane of oscillations of the incident light and the plane of the polarizer, will pass through the device. Hence the intensity of the transmitted light I (remine that $I \sim A^2$) is determined by expression

$$\mathrm{I} = \mathrm{I}_0 \cos^2 \varphi. \qquad (6.14)$$

Equation (6.14) is called the law of Malus.

Polarization in reflection and refraction. If the angle of incidence on the interface between two dielectrics differs from zero, the reflected and refracted rays will be partly polarized. Oscillations perpendicular to the plane of incidence predominate in the reflected ray (in Fig.6.9 these oscillations are denoted by points) and oscillations parallel to the plane of incidence predominate in the refracted ray (they are depicted in the figure by double headed arrows). The degree of polarization depends on the angle of incidence.

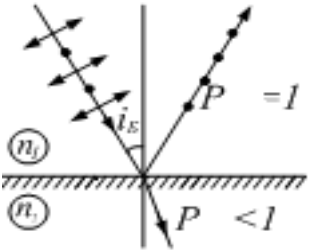


Figure 6.9 – Brewster's Law

At an angle of incidence $i = i_B$ the reflected ray is completely polarized (it contains only oscillations perpendicular to the plane of incidence).

The angle i_B is determined by relation

$$tg(i_B) = \frac{n_2}{n_1},$$

which is known as Brewster's law, and angle i_B is called Brewster's angle.

Polarization in Double Refraction. When light passes through all transparent crystals except for those belonging to the cubic system, a phenomenon is observed called double refraction. It consists in that a ray falling on a crystal is split

inside the latter into to rays propagating, generally speaking, with different velocities and in different directions.

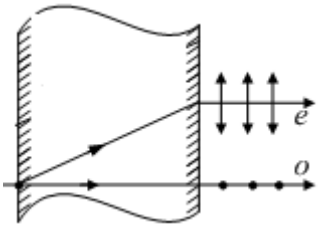


Figure 6.10 – Birefringence in an anisotropic crystal

Doubly refracting crystal, a divided uniaxial and biaxial one. (In the following, we shall be concerned only with uniaxial crystal). In uniaxial crystals, one of the refracted rays obeys the conventional law of refraction, in particular it is in the same plane as the incident ray and a normal to the refracting surface. This ray is called an *ordinary* ray and is designated by the symbol *o*. For the other, called an *extraordinary* one (designated by *e*), the ratio of the sines of the angle of incidence and the angle of refraction does not remain constant when the angle of incidence varies. Even upon normal incidence of light on a crystal, an extraordinary ray, generally speaking, deviates from a normal (see. Fig.6.10). In addition, an extraordinary ray does not lie, as a rule, in the same plane as the incident ray and a normal to the refracting surface. Examples of uniaxial crystals are Iceland spar, quartz and tourmaline.

Uniaxial crystals have a direction along which ordinary and extraordinary rays propagate without separation and with the same velocity. This direction is called the optical axis of the crystal. It must be born in mind that an optical axis is definite direction in the crystal. Any straight line parallel to the given direction is also an optical axis of the crystal.

CHAPTER 7

QUANTUM OPTICS

7.1 Thermal radiation

Radiation of electromagnetic waves by heated bodies is called thermal radiation. The energy source of thermal radiation is an internal energy of the body. Thermal radiation is the single type of radiation that can be in equilibrium with the radiating body, i.e. the amount of radiated energy per unit of time equals to the amount of absorbed energy. To characterize the thermal radiation are used the next quantities:

1. Energy flux – the amount of energy emitted per unit time. Instantaneous energy flux is the first time derivative of the energy emitted by the body

$$\Phi_e = dW/dt. \qquad (7.1)$$

2.Integral body radiation density R_e – a flux of energy, which is radiated by unit body surface area in all the range

$$R_e = d\Phi_e/ds. \qquad (7.2)$$

3. Introduce dR_ω – the flux of energy emitted by unit surface in the frequency range $d\omega$. This value is proportional to the frequency interval $d\omega$

$$dR_\omega = r_\omega d\omega. \qquad (7.3)$$

The proportionality factor r_ω is called the spectral density of radiation of the body

$$r_\omega = dR_\omega/d\omega. \qquad (7.4)$$

It is flux of energy per unit surface in unit frequency interval.

In view of formula (7.4). Integral density of radiation of the body is determined by the formula

$$R_e = \int dR_\omega = \int r_\omega d\omega. \qquad (7.5)$$

4. Absorption capacity of the body $a\omega$ equals the ratio of absorbed flux $d\Phi'\omega$ to the incident flux $d\Phi\omega$ body in the same frequency range $d\omega$

$$a_\omega = d\Phi'_\omega / d\Phi_\omega . \quad (7.6)$$

If a a_ω= 1, i.e. the body absorbs all radiation falling on it. Such body is called *total radiate* or *absolutely black body* or simply *black body*.

The best black body model is the inner cavity in the body with a small hole, which the internal surface is covered by soot. A beam of light penetrating such a cavity through a hole is reflected many times and is completely absorbed.

7.2 Laws of thermal radiation

Kirchhoff's law. The ratio of emissivity $a_{\omega,T}$ to absorptivity $r_{\omega,TT}$ does not depend on the nature of the body and equals the universal function of frequency and temperature $f(\omega,T)$

$$r_{\omega,T} / a_{\omega,T} = f(\omega,T). \quad (7.7)$$

For a black – body (marking its parameters by star) $a^*_{\omega,T} = 1$ and therefore

$$f(\omega,t) = r^*_{\omega,T} ,$$

i.e., a universal function of the Kirchhoff $f(\omega,t)$ equals the spectral density of black body radiation . Then for the integrated density of blackbody radiation we get

$$R^*_e = \int f(\omega) d\omega . \quad (7.8)$$

Graphs of Kirchhoff`s function depending on the wavelength of radiation for different temperatures of body are shown in Fig. 7.1.

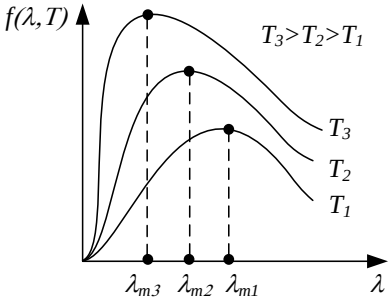


Figure 7.1 – Graphs of the Kirchhoff function depending on the wavelength of radiation for different body temperatures

The transition from $f(\omega,t)$ to $f(\lambda,t)$ may be made by the formula

$$f(\omega,t)=2\pi c\, f(\lambda,t)/\omega^2.$$

At thermal radiation, the full range of electromagnetic waves of lengths from 0 to ∞ emitted at the same time. But the waves of different lengths carry a different energy. Wavelength, which carries a maximum energy (which corresponds to the maximum of $f(\lambda,t)$, is denoted by λ_m.. With increasing temperature the maximum $f(\lambda,t)$ is shifted into the area of shorter wavelengths. That explains the change in color of the metal body at heating from crimson to white.

Stefan – Boltzmann's law. The integrated density of blackbody radiation is proportional to the fourth power of the absolute temperature

$$R_{e*} = \sigma T^4,$$

where $\sigma = 5{,}7\cdot10^{-8}$ (W/m^2K) – is the Stefan-Boltzmann's constant.

Wien's Law. With increasing, of temperature the wavelength λ_m, which corresponds the maximum of the spectral distribution $f(\lambda,T)$ decreases, but the product $\lambda_m T$ remains constant

$$\lambda_m T = b,$$

where $b = 2{,}9\ 10^{-3}$ (mK) – is Wien's constant.

7.3 Planck's formula

In 1900 Planck obtained the formula of function $f(\omega,T)$. It has the next view

$$f(\omega,t)=\frac{\hbar\omega^3}{4\pi^2c^2}\frac{1}{e^{\hbar\omega/kT}-1}, \qquad (7.9)$$

Calculations done according by the formula (7.9) agree with experimental results. Planck made an assumption that was completely strange to classical conceptions: electromagnetic radiation may radiate by separate portions (quantum) of energy, the magnitude of which is proportional to the frequency

$$\varepsilon_n=\hbar\omega,$$

where $\omega = 2\pi\nu$ – cyclic frequency (ν – frequency)**,**
$\hbar=h/2\pi$,, h – Planck's constant *($h=6,62\cdot10^{-23}$ J s)***.**
If the radiation is generated by portions $\hbar\omega$, then its energy ε_n must be a multiple quantum energy

$$\varepsilon_n=n\hbar\omega \qquad (n=0,1,2,\ldots).$$

Average time value of radiation energy of frequency ω is defined by the formula

$$<\varepsilon>=\sum P_n\,\varepsilon_n=\frac{\hbar\omega}{e^{\hbar\omega/kT}-1}. \qquad (7.10)$$

When $\hbar \to 0$ formula (7.10) transforms in classic formula of the average energy of electromagnetic oscillations $<\varepsilon> = kT$.
From formula (7.10), Wien's and Stefan-Boltzmann's laws can be obtained. So, Planck's formula (7.10) gives an exhaustive description of equilibrium thermal radiation.

Photoeffect. Photons. Emission of electrons from the metals owing to the action of light is called photoeffect or photoemission.

In 1905 A. Einstein worked out the theory of photoeffect, that is based on corpuscular nature of light. This theory gives just qualitative and quantitative explanation of

experimental data. In accordance with presentations of Einstein, light is propagated and is absorbed by the same portions (quantum). Coming from supposition about existence of light quantum, Einstein got equation of photoeffect

$$\hbar\omega = A + \frac{mv_{max}^2}{2}$$

In according with this equation the energy of the incident photon $\hbar\omega$ is expended to electron work of exit of the metal A and giving it kinetic energy.

$$E_к = \frac{mv_{\max}^2}{2}\ .$$

The least energy what is necessary to provide an electron to remove it from a solid or liquid in a vacuum is called work of exit. The photon, like any particle has energy and momentum.

The energy of the photon $\varepsilon = \hbar\omega$. The momentum of photon

$$p = \frac{\hbar\omega}{2\pi c} = \hbar k.$$

Compton effect is the phenomenon of change of electromagnetic radiation wavelength λ (*X*- and gamma -rays) in the scattering on free or weakly bounded electrons of substance.

Compton experiment shows that in the scattered radiation with the primary wavelengths λ there is also rays of longer wavelength λ'. To describe the Compton effect, it can consider the interaction of particles as elastic collision in which the laws of conservation of energy and momentum are executed. Thus are obtained two equations, the solution of which gives

$$\Delta\lambda = \lambda' - \lambda = \lambda_c(1-\cos\theta),$$

where - $\lambda_c = 2\pi\hbar/mc$, θ - diffraction angle of photon.

PART 3

QUANTUM MECHANICS. ELEMENTS SOLID STATE PHYSICS. NUCLEAR PHYSICS

CHAPTER 8

QUANTUM MECHANICS

8.1 De Broglie hypothesis

In the phenomena of radiation and absorption of light, the photoelectric effect the properties of light particles (corpuscles) are manifested. Thus it can be argued that light has corpuscles – wave nature.

The wave properties of matter. In 1924, de Broglie proposed the hypothesis that the dual nature is inherent not only in light, but also in all microparticles.

A photon has energy and momentum

$$\varepsilon=h\nu, \qquad p=h/\lambda.$$

Energy and momentum reflects its particle properties.

According to the de Broglie's idea a movement of any particle associated with the propagation of a plane wave

$$\psi = ae^{-i(\omega t-kx)}, \tag{8.1}$$

whose length $\lambda=h/p$ and frequency $\nu=\varepsilon/h$.

These waves are called initially de Broglie's wave, and then it was called matter waves. Hypothesis de Broglie was soon confirmed in experiments with the passage of electron beam through a metal foil (Fig. 8.1 a).

Accelerated electron beam passes through a thin metal foil and hits the photographic plate (PP). Electrons hitting photographic plate makes it the same effect as the photons, i.e. causes it lighting up. Electron beam is scattered by metal foil and gives diffraction pattern. similar one that occurs in the fall of X-ray

(Fig. 8.1, b). In this case, diffraction pattern corresponds to a electron's wavelength $h / mv, = \lambda$, where v – velocity of electrons.

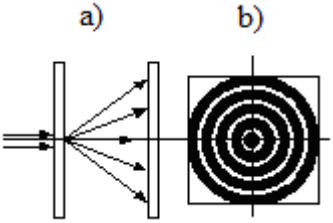


Figure 8.1 – a) Passage of an electron beam through a metal foil; b) an accelerated electron beam passes through a thin metal foil and strikes a photographic plate

Thus, the matter is simultaneous and particle and wave properties.

8.2 Schrödinger equation

Due to the fact that particles have wave properties, there is a need to build mechanics that takes into account the wave properties of particles, so equation of this mechanics must be of a wave equation. This mechanics is called quantum mechanics. The basic equation of quantum mechanics is the Schrödinger equation

$$-\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x^2}+U\psi = i\frac{\partial^2\psi}{\partial t^2},$$

where m – mass of the particle;

U – potential energy of a particle in a force field in which it moves;

$\psi = \psi(x,t)$ – the wave function.

Equation (8.2) is a one-dimensional temporal Schrödinger's equation. For stationary states, i.e. states that do not change on time, the Schrödinger equation has the form

$$\frac{d^2\Psi}{dx^2}+\frac{2m}{\hbar^2}(E-U)\Psi=0, \qquad (8.2)$$

where E – total energy of particle;

$\Psi = \Psi(z)$ – the wave function, depending on the coordinates and time and determining the state of the particle.

The physical content of the wave function is that the square of its module determines the probability density of finding the particle at the corresponding place of space. Then the probability dP that the particle is founded within the volume dV, is defined by the formula

$$dP = |\psi|^2 dV .$$

Uncertainty relation. Quantum mechanics has statistically character. It can not determine the location of a particle in space or path along which the particle moves. The such concepts as a particular location and trajectory of microparticles are absent, as movement along a particular path has not compatibility with the wave properties of particles. Degree of accuracy with which an idea of a certain position in space can be applied to a particle is given the uncertainty relation. According to this relation the particle can not have simultaneously exact coordinates and the corresponding value of its momentum. The mathematical formulation of this principle has the form

$$\Delta x \cdot \Delta p_x \geq \hbar,$$

where Δx – uncertainty (absolute error) of coordinate;

ΔP_x – uncertainty of momentum along that coordinate.

From the uncertainty relation implies that the more accurately is determined one of the values (coordinate or momentum along this coordinate), the greater the uncertainty (error) of another variable.

Quantization of energy. According to its physical content the wave function ψ (x,y,z) must be single-valued, finite and continuous throughout into the domain of definition

x, y, z. Schrödinger equation has a solution that satisfies this condition is not under any values of the energy E, but only under certain defined values, which are called eigenvalues for a particular quantum system.

Let us determine the eigenvalues of energy and corresponding eigenfunctions for a particle that moves in a one-dimensional infinitely deep potential well along the x axis (Fig. 8.2).

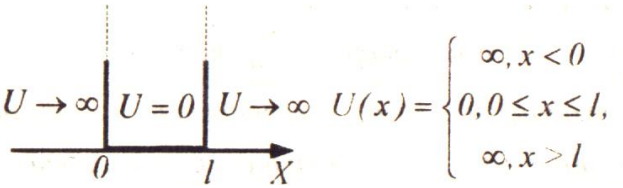


Figure 8.2 – One-dimensional infinite potential well

The movement of particle is limited impermeable the wall. The potential energy U is zero inside the well and turns into infinity outside it when $x \le 0$ and $x \ge l$. Inside the well $U = 0$ and the function must satisfy the equation

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2} E\Psi = 0 .$$

Outside the well $U = \infty$ and $\Psi = 0$. The solution of differential equation of the second order is function *sin* or *cos*. We write the solution in the form

$$\Psi = a \sin kx, \qquad (8.3)$$

where – $k^2 = \frac{2m}{\hbar^2} E$.

Satisfying conditions of continuity of Ψ – function on the boundaries x=0 and x=l

$$\Psi(0) = 0 \quad , \qquad \Psi(l) = 0$$

will obtain eigenvalues of energy of the particle

$$E_n = \frac{\pi^2 \hbar^2}{2ml^2} n^2, \qquad (n=1,2,3,\ldots). \qquad (8.4)$$

Thus, the energy of a particle in a given force field is quantized, i.e. it is changed discretely by quanta (portions) of energy). Other characteristics (momentum, etc.) is quantized also.

8.3 Atom hydrogen

In an atom the electrons are in a three-dimensional potential box. The problem of electron motion in three-dimensional potential box can be solved based on the solution of one-dimensional problems, using the method of division of variables. The Ψ-function $\Psi = \Psi(x,y,z)$ is represented as the product $\Psi = \Psi_1(x)\,\Psi_2(y)\,\Psi_3(z)$ and three-dimensional problem is decomposed into three independent one-dimensional one. The energy of the atom. In the hydrogen atom the potential energy

$$U = -Ze^2/r,$$

where e – electron charge;
r – distance of the electron from the nucleus;
Z – integer.

If $Z = 1$ – it is a hydrogen atom, and when $Z \neq 1$ – hydrogen – like atom, i.e. an atom of any element which gave up all electrons except one.

Schrödinger equation in three-dimensional (spherical) system coordinates for stationary states of hydrogen-like atom has the solution in subsequent cases:

1) If electron energy E. has any positive values. This case corresponds to a free electron, flying near the nucleus and goes back to infinity (Fig.8.3).

2) If electron energy E. has the discrete negative values. This case corresponds to electrons that are inside the atom. The energy of electron is determined with the formula

$$E_n = -me^4 z^2 / 2\hbar^2 n^2, \qquad (n = 1,2,3 \ldots).$$

Quantum numbers. The wave functions corresponding to the values of the energy E_n, contain three integer parameters:

1) n – principal quantum number, which takes integer values $n = 1, 2, 3 ...$;

2) l – azimuthal quantum number, which takes integer values $l = 0,1,2,3, ..., (n – 1)$;

3) m – magnetic quantum number, which for a given azimuthal number l takes integer values m – l, ..., 0, ... + l. – in all $(2l + 1)$ meanings.

Thus, each value of the energy E_n corresponds to several wave functions , which have different quantum numbers l and m. The states with the same energy are called degenerates. Thus the states with the same value n are n^2 – fold degenerate

$$\sum_{l=0}^{n-1}(2l+1) = n^2 . \qquad (8.5)$$

Azimuthal quantum number l determines the orbital that is associated with the motion in its orbit around the nucleus, the angular momentum of an electron in an atom. Law of quantization of angular momentum of the electron

$$M_e = \hbar\sqrt{l(l+1)} .$$

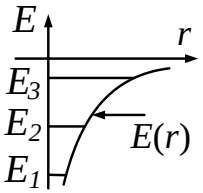


Figure 8.3 – Potential energy and energy levels of a bound state

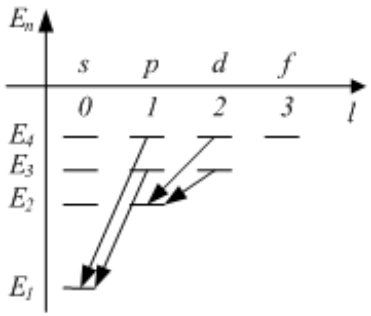


Figure 8.4 – Energy levels and electronic transitions in an atom

The magnetic quantum number m determines the projection of the orbital angular momentum onto the vector magnetic field. The correspondent quantization law

$$M_B = m\hbar.$$

Thus, the orbital angular momentum of an electron in an atom and its projection on the direction of the magnetic field, as energy are discrete.

In atomic physics the state of an electron in an atom with different azimuthal quantum number denoted by letters.

l = 0 – *s*- state,
l = 1 – *p*- state,
l = 2 – *d*- state,
l = 3 – *f*- state.

Then *g, h* and beyond the Latin alphabet. For example, if the state of the electron 3*p*, it means that n = 3, l = 1.

Scheme of allowed energy (energy spectrum) can be conveniently represented as that shown in Fig. 8.4.

The emission and absorption of energy occurs when hydrogen atom electron makes transition from a state with a value of energy E_n in the state at another energy (in short – when electrons from one energy level makes transition to another one).

There are only those transitions for which the azimuthal quantum number l changes to the unit: $\Delta l = \pm 1$.. It is a consequence of the law of conservation of angular momentum in the emission and absorption of photon.

Emission and absorption spectra. Transitions $np \rightarrow 1s$ $(n = 2,3 ...)$ form a series of emission lines called the Lyman's series. Transitions ns $ns \rightarrow 2p$, $nd \rightarrow 2p$, $np \rightarrow 2s$ $(n=3,4...)$ form a series of emission lines , called the Balmer's series. State 1s – state of minimum energy is called the *ground* state.

Atom absorbs (and emits) only those photons whose energy corresponds exactly to the energy difference between its two levels.

8.4 Many-electron atoms

In atoms, containing several electrons, each electron is moved in the averaged field of the nucleus and the remaining electrons. The electronic energy level in a central force field depends not only on the principal quantum number *n*, but also on the azimuthal number l .

The angular momentum of the atom as a whole consists of angular momentum element of electrons that make up the atom. Transitions in the energy level diagram of an atom submits to the following selection rule : only such passages, in which the angular momentum of the atom changes by one are possible: $\Delta M = \pm 1$**.**

Electron spin. If the atoms emitting light are in a magnetic field, the emission line in the spectrum of radiation of an atom is split into two separate lines. To explain the structure of the spectrum in 1925 has been enlisted hypothesis that the electron has a own angular momentum M_s is not associated with the movement of electron along its orbit. This momentum was named – *spin*. Spin should be considered as an intrinsic property peculiar to electron, just as the charge and mass. The value of its own angular momentum of microparticle is defined by quantum number s:

$$M_s = \hbar\sqrt{s(s+1)}\,.$$

Where S $= 1/2;\ M_s = \hbar\sqrt{3}/2$**.**

Spin projection to the direction of the magnetic induction vector $\vec{B}$

$$M_{S_B} = m_s\hbar\,,$$

The total angular momentum of electron in atom M_j is a sum of orbital where electron spin quantum number $m_s = \pm s = \pm 1/2$**.**

M_e and spin M_s angular momentums:

$$M_j = M_e + M_s\,.$$

Mechanical moments cause magnetic moments which interact with each other just like two interacting contours with currents. The energy of this interaction (called spin -orbit interaction) depends on the relative orientation of the spin and orbital moments.

8.5 The distribution of electrons in the atom for energy levels

Thus, the state of an electron in an atom is characterized by *four quantum numbers:*

1) general n = 1,2,3 ... (any integer);

2) azimuthal l = 0,1,2, ..., n–1 – n total values;

3) magnetic m_l = – l, ..., 0, ... + l – Only $2l$ +1 values;

4) spin $m_s = \pm 1/2$ only two values.

Energy is mainly determined by the quantum numbers n and l. In addition, there is a weak dependence of m_l and m_s.

The distribution of electron states with different energy, subjects to two principles:

1. Pauli principle. At the same atom no two electrons with the same set of four quantum numbers n, l, m_l, m_s**.**

2. The principle of minimum energy. In the unexcited state of atom electrons should be placed at the lowest allowed energy levels for them.

This state of the atom is called ground state. Energy levels differ of one another by quantum numbers – n, l, m_l. Then at each level (i.e., in a state with a same given energy) can be only two electrons with opposite oriented spins. The set of electrons of an atom that have the same meaning of quantum number n, form *shell*. Number of electrons in the shell.

$$N_{об} = 2n^2. \qquad (8.6)$$

The totality of electrons with the same principal quantum number n, but different values of l forms a *subshell*. Number of electrons in subshell

$$N_{no} = 2(2l+1) \ .$$

According to (8.6) in the shell with n = 1 may also be located two electrons in the second – 8, in the third, -18, etc. That corresponds to the number of elements in the periodic table of elements, i.e. Pauli's principle explains this number by maximum number of electrons that can be in regular shell.

CHAPTER 9

ELEMENTS SOLID STATE PHYSICS

The subject of this chapter is to study the properties of solids, which are caused by the movement of electrons – conductivity of solids.

9.1 Distribution of Electrons by Energies. Fermi Energy

As already known, distribution of electrons in an atom is submits to Pauli's exclusion. When atoms are combined in the crystal it can be considered as the giant molecule. In such molecule all electrons of atoms are common and can be regarded as a single quantum mechanical system. States of electrons in the crystal is also are described by four quantum numbers, but another than in the atom:

$$n_x, n_y, n_z = 0, \pm 1, \pm 2 \ldots, \qquad m_s = \pm \frac{1}{2} \ .$$

Electrons of conduction in the metal can be considered as an ideal gas that can be described by the function of the Fermi- Dirac distribution. The average number of electrons in the quantum state at the energy level with energy E is,

$$w = \frac{1}{e^{\frac{E - E_F}{kT}} + 1}$$

where E_F – Fermi level the highest energy level occupied by electrons at absolute zero temperature;
w – occupation probability of a given level.

At a temperature T = 0 K, electrons occupy all the levels from the lowest to the Fermi level, and states with E $> E_F$ are free.

9.2 Splitting of energy levels. Emergence of energy bands

At the considering a system consisting of a large number of atoms the main theoretical ideas remain unchanged. First of all, make the following conclusion: in a system consisting of N atoms, the number of quantum states must be N times greater than in the free atom, only in this case it is possible to satisfy the Pauli principle.

Spectral studies suggest that significant changes concern only the external valence electrons. Thus, the quantum state of solids is closely associated with the quantum state of the atom. Bond quantum states of atoms and bodies are expressed the following right: body of N atoms has N times greater energy states than a single atom. Periodic electric field of the crystal grating, the interaction between atoms significantly affect the energy levels of electrons in solids. The result of this effect is the splitting of the electron levels. Instead of a single level identical for all *N* isolated atoms in solids *N* closely spaced levels which form a strip or band of allowed energy levels appears (Fig. 9.1).

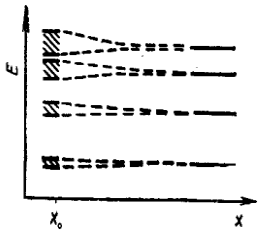


Figure 9.1 – Formation of energy bands in solids

Emergence of the energy bands. Physical properties of crystals are determined mainly by the upper bands, which have yet electrons. The energy spacing between the "bottom" (minimum energy) of the upper band, in which are still the electrons and

the "ceiling" (maximum energy) prior to filled band called the forbidden band. Allowed band in the solid can be filled completely or partly or be free. Depending of that, the solids are divided into conductors, semiconductors and insulators.
If at $T = 0$ all bands in which there are electrons are completely filled, and the next "empty" allowed one is separated from the nearest allowed band of wide forbidden band (width of the forbidden band $\Delta E \sim 3\ eV$) crystal is a dielectric (Fig. 9.2), if $\Delta E \leq 1\ eV$ – semiconductor.

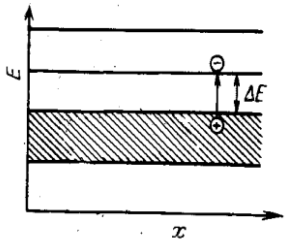


Figure 9.2 – Energy bands in solids (conduction, valence, and forbidden band)

Last completely filled band is called the valence band, the following allowed one – the conduction band – the band of free electrons.

If the upper band in which there are electrons are filled partially $(\Delta E = 0)$, , then this is a conductor, usually a metal, such as Li, Na, and others (Fig. 9.3).

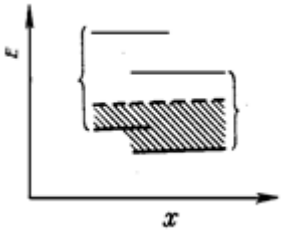


Figure 9.3 – Partially Filled Band in Metals

For some typical metals there are overlapping of allowed bands (Fig. 9.4).

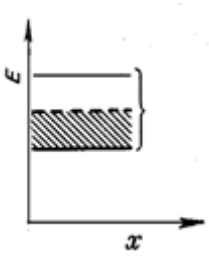


Figure 9.4 – Overlapping bands in metals

CHAPTER 10

NUCLEAR PHYSICS

10.1 Atomic nucleus. The composition and characteristics of the atomic nucleus

The nuclei of the atoms consist of two types of fundamental particles protons and neutrons (their common name nucleon). Proton (denoted by p) has a mass

$$m_p = 1836 m_e$$

and the charge $q_p = e = 1{,}6 \cdot 10^{-19}$ C (elementary charge).
Neutron (denoted by n) has mass

$$m_n = m_p + 2{,}5 m_e$$

and its charge. $q_n = 0$.
Characteristics of the atomic nucleus:
1. Z – atomic number or charge number equal to the number of protons determined the nuclear charge (+ Ze);
2. N – number of neutrons in the nucleus;
3. A – mass number (atomic mass) equal to the number of nucleons

$$A = Z + N.$$

Symbol of the nucleus ${}^{A}_{Z}X$. Most chemical elements have several varieties – isotopes differing by values of mass number A. For example, isotopes of hydrogen: ${}^{1}_{1}H$ – ordinary hydrogen (p), ${}^{2}_{1}H$ – deuterium (d), ${}^{3}_{1}H$ – tritium (t).Isotopes of oxygen, ${}^{16}_{8}O$, ${}^{17}_{8}O$, ${}^{18}_{8}O$.
Thus isotopes are kernels with the same number of protons. Nuclei with the same mass number A are called *isobars*, for example (${}^{40}_{18}Ar$, ${}^{40}_{20}Ca$). Nuclei with the same number of neutrons are called *isotone*, for example (${}^{13}_{6}C$, ${}^{14}_{7}N$). The radius

of the nucleus with sufficient accuracy is given by r = 1,2 $A^{1/3}$ 10^{-15} *m*.

The volume of the nucleus is proportional to the number of nucleons in the nucleus.

Mass Defect and Separation Energy of the Nucleus. The mass at rest of the nucleus always less than the sum of the masses of particles that make up the nucleus. Energy equal to the work done to separate nucleons, which form the nucleus is called *binding energy* E_b. Thus the energy of the nucleus is less than the energy of non-interacting nucleons Reducing energy ΔE must be accompanied by an equivalent decrease in mass $\Delta m = \Delta E/c^2$. Consequently, the separation energy of the nucleons in the nucleus

$$E_b = c^2\{[Zm_p+(A–Z)\ m_n]–m_{nucleus}\}. \qquad (10.1)$$

For example for helium ${}^4_2He$

$$E_b=28,4\ MeV\ (1\ eV=1,6\cdot10^{-19}\text{J}).$$

Its increasing mass number *A* specific separation energy (E_b/A) decreases (see Fig. 10.1).

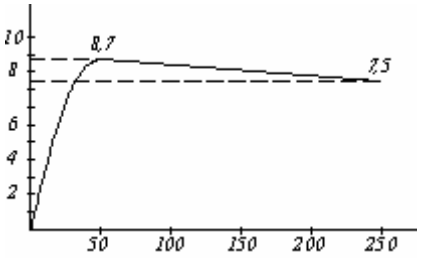


Figure 10.1 – Example of a saturation curve or dependence with saturation

For uranium (A = 240), it is 7.5 MeV/*nucleon*. Stronger than all the nucleons connection in nuclei is one with mass numbers 50-60, E_s/A=8,7 MeV/*nucleon*.

$$\frac{E_s \ \ MeV}{A \ nucleon}$$

This dependence of the *specific binding energy* of the mass number makes energetically favorable (i.e., with the release of energy) two processes as:

a) the fission of heavy nuclei into several lighter ones;

b) the fusion of light nuclei into one.

Both of these processes should be accompanied by the release of a large amount of energy. For example, the division of a single nucleus with mass number *A = 240* (specific separation energy of 7.5 MeV) into two nuclei with mass numbers

A = 120 (specific separation energy 8.5 MeV) leads to the release of approximately 240 MeV of energy.

At the confluence of two nuclei of heavy hydrogen into helium nucleus energy 24 MeV should be released, 6 MeV for one nucleon. Thus confluence of light nuclei 6 times more profitable for the separation of heavy nuclei.

For comparison, recall that when confluencing one carbon atom with two oxygen atoms releases energy ~ 5 eV, at the separation of heavy nuclei, or confluence of light nuclei releases energy millions of times over.

The tremendous separation energy of the nucleons in the nucleus suggests that between nucleons in the nucleus there is a very intensive interaction. This interaction has nature of attraction and keeps the nucleons at a distance $\sim 10^{-15}$ m apart.

Nuclear interaction between nucleons is called the strong interaction**.**

10.2 Radioactivity

Radioactivity it is spontaneous transformation of unstable isotopes of a chemical element to another, which is accompanied by radiation of elementary particles and nuclei.

The major of such changes include: 1) α- decay, 2) β- decay, 3) proton radioactivity, 4) spontaneous fission of heavy nuclei.

Radioactivity of natural isotopes observed in the nature is called natural one. Radioactivity isotopes obtained by nuclear reactions is called artificial one. Between natural and artificial radioactivity is no fundamental difference. The process of radioactive transformations in both cases are governed the same laws.

Each nucleus is a probability λ that the nucleus undergoes conversion per unit time. So if a radioactive substance contains the N atoms, the number of atoms dN who experience the transformation in a time dt, is proportional to the number of available nuclei and the interval of time

$$dN = -\lambda N\, dt. \qquad (10.2)$$

Integrating (10.2), we obtain

$$\ln N = -\lambda t + const,$$

from which implies the law of radioactive decay:

$$N = N_0 e^{-\lambda t},$$

where N_0 – number of nuclei that have not decayed at the initial time, N – number of nuclei that have not decayed at time t, λ – characteristic constant for a radioactive substance, called a *disintegration constant*. Thus, the number of radioactive atoms decreases with time exponentially.

Number of nuclei that decayed by time t is given by

$$\Delta N = N_0 - N = N_0\left(1 - e^{-\lambda t}\right).$$

Time during which half the original number of atoms is decayed called a *half-life* T. The value of T is defined by condition

$$\frac{1}{2} N_0 = N_0\, e^{-\lambda T}$$

from which

$$T = \frac{\ln R}{\lambda} = \frac{0{,}693}{\lambda}$$

Half-life period for known nowadays radioactive substance has ranges from $3 \cdot 10^{-7}$s to $5 \cdot 10^{15}$s. It may happen

that the nuclei arising at radioactive transformation of the original nucleus will be radioactive and also decay. New products of decay can also be radioactive. As a result, there are a number of radioactive transformations. In nature, there are radioactive series, fathers which are ^{238}U (uranium series), ^{232}Th (thorium series), ^{235}U (actinium series). The final product in all three cases are isotopes of lead.

10.3 Nuclear reactions

Fission of nucleus. In 1938, German scientists Hahn and Strassman discovered that by irradiating uranium, neutrons produced elements barium and lanthanum. This is because the uranium nucleus, upon capturing the neutron, undergoes division into two parts, which are called fission fragments. Separation can take place in various ways, but more probably is the division into fragments, which mass relate as 2 to 3. Binding energy, resides for one nucleon of the radioactive products is significant higher than in heavy nuclei. It implies that the separation of heavy nuclei accompanied by release a large amount of energy. It was also important that the separation of each nucleus were released a few neutrons. On average, 2.5 neutrons reside on one act of division. One of the ways on which the division is as follows:

$$^{235}_{92}U + n \rightarrow ^{236}_{92}U \rightarrow ^{140}_{55}Cs + ^{94}_{57}Rb + 2n.$$

Nuclei ^{235}U and are divided by neutrons of any energy, but especially good – by slow neutrons. Nuclei ^{238}U is divided by only fast neutrons (with energies of at least 1 M*eV*). At lower energies the neutrons are absorbed by nuclei without their separation. This process is called radioactive abortion. The emergence of neutrons at the separation of nuclei^{235}U, ^{233}U makes possible to realize nuclear chain reaction. Indeed, *z* neutrons that were born at the fission of one nucleus can cause separation of *z* nuclei and in the result z^2 new neutrons will be

born that cause nuclear division z^2 nuclei, etc. Thus, the number of neutrons that are born in each generation increases in geometry progression. Neutron multiplication in the fusser runs very fast. Owing to limited size of body and great penetrating power of neutrons, many of them leave the reaction zone before than they will be captured. The relative proportion of neutrons emitted from the body decreases with increasing mass. Natural uranium contains 99.3% isotope ^{238}U and 0.7% of the isotope ^{235}U. Thus, for each nucleus, which is divided under the influence of slow neutrons resides 140 nuclei that capture not too fast neutrons without fission. Therefore, in the natural state, a chain fission does not arise. Nuclear chain fission in uranium can be done in two ways. The first involves the separation of uranium isotope ^{235}U, which is divided by slow-neutrons.

In a piece of pure ^{235}U each neutron captured by the nucleus realizes the division with birth on average 2.5 new neutrons. However, if the weight of the piece is less than a certain critical value, the majority of births neutron flies out without causing division, so that the chain reaction does not occur. When mass is more *critical*, neutrons multiply rapidly and explosion takes place.

10.4 Nuclear reactors

Another way to make a governing chain fission is used in nuclear reactors. In the reactor is used natural (or slightly enriched isotope) of uranium ^{235}U. To prevent radiative capture of neutrons by uranium ^{238}U, relatively small blocks (pieces) of the dividing substance are placed at some distance from each other. The gaps between the blocks are filled by *moderator*, i.e. a substance which decelerate neutrons to thermal velocities.

The deceleration of neutrons is carry out due to the elastic scattering. In this case, the energy expended by particle,

depends on the kind of the colliding particles. The maximum amount of power is consumed if the the particles have the same masses. In slow – neutron reactors is used fuel enriched biotope (^{235}U or ^{239}Pu).

In Fig. 10.2 a scheme of the low reactor is shown. In figure it is marked by number 1 – moderator (graphite), 2 – blocks of uranium, 3 – cores that contain cadmium or beryllium, 4 – biological defense. These rods -3 are used to regulate the process in the reactor. Cadmium and beryllium absorbs neutrons. So putting the rods into the reactor reduces neutron multiplication factor, their output – increases. Automatic device that manages Fig. allows the reactor to maintain a well-developed capacity for a given level.

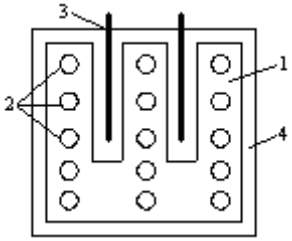


Figure 10.2 – Schematic representation of electrical conductivity measurement using electrodes

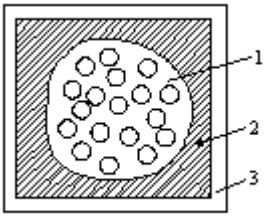


Figure 10.3 – Model of a conductive region formed by conductive particles embedded in an insulating matrix

Scheme of reactor on fast neutrons , is shown in Fig.10.3, where 1 – active zone of the plutonium rods, 2 – natural uranium ^{238}U, 3 – biological defense. The plutonium nuclei are separated well by fast neutrons and sustain a chain reaction, because if not need in the moderator. In natural uranium neutrons, emitted from the active zone, step into reaction

$$^{238}U + n \rightarrow ^{239}U + \gamma,$$

that convert uranium into plutonium.

10.5 Thermonuclear reactions

Synthesis of nuclei, i.e. merger of light nuclei into one nucleus is accompanied by release of a high quantity of energy. As for nuclear fusion it is necessary high temperature, a process is called thermonuclear reaction.

To overcome the potential barrier due to Coulomb's repulsion, nuclei with atomic numbers Z_1 and Z_2 should have the energy

$$E = \frac{Z_1 Z_2 e^2}{r},$$

where r – the radius of action of the nuclear forces , which is ~ $2 \cdot 10^{-15}$ m. Even for the lightest nuclei with $Z_1=Z_2=1$, this energy is $E = 0.7$ *MeV*. The average energy of thermal motion equal to *0.35 MeV* (according the formula $E = kT$) corresponds to temperature ~ $2\ 10^9$ K. • However, the synthesis of light nuclei can occur at much lower temperatures. Owing to the distribution of particles by velocities there is some quantity of nuclei with significantly higher energies than average. Therefore, some thermonuclear reactions can occur at the temperatures ~10^7 K.

The easiest way to create the conditions for the synthesis of nuclei of deuterium and tritium:

$$^2_1H + ^3_1H \rightarrow ^4_2He + n.$$

This reaction is accompanied by the release of energy at 17.6 MeV, which is ~ 3.5 MeV per nucleon. For comparison, we note that the separation of uranium results in the release of ~ 0.85 MeV per nucleon.

For the realization of controlled thermonuclear reactions, it must attain and maintain some volume at the temperature $\sim 10^8$ K (100 million degrees). At such high temperatures, the substance is a completely ionized gas – plasma. Along with the need to obtain extremely high temperatures, there is a problem of the confinement of plasma in a given volume. Touching of plasma to the walls of volume bring to its cooling. In addition, the wall of any substance at a such high temperature immediately evaporates.

To thermonuclear reaction will be proceeding with great release of energy, we must also have the necessary concentration of deuterium and tritium nuclei per unit volume (i.e. the desired density of plasma). These two quantities are interrelated: the higher the concentration of atomic nuclei, the less time is needed to maintain the plasma and vice versa. This dependence is determinate by Lawson criterion.

For the deuterium – tritium reaction (D + T-reaction) and $T=10^8$ K Lawson criterion is $3 \cdot 10^{14}$. This means that when the concentration of nuclei $3 \cdot 10^{14}$ g/cm3 plasma confinement time should be ~ 1 s.

REFERENCE

1. Орел Є. С. Курс фізики : навч. посіб. / Є. С. Орел, А. В. Безуглий, О. М. Петченко, Є. І. Назаренко. – Харків: ХНУМГ ім. О. М. Бекетова, 2018. – 191 с.
2. Кучерук І. М. Загальний курс фізики : навч. посіб. / І. М. Кучерук, І. Т. Горбачук, П. П. Луцик. – Т. 1. – Київ: Техніка, 1999. – 536 с.
3. Кучерук І. М. Загальний курс фізики : навч. посіб. / І. М. Кучерук, І. Т. Горбачук, П. П. Луцик. – Т. 2. – Електрика і магнетизм. – Київ: Техніка, 2000. – 512 с.
4. Кучерук І. М. Загальний курс фізики : навч. посіб. / І. М. Кучерук, І. Т. Горбачук, П. П. Луцик. – Т. 3. – Оптика, квантова фізика. – Київ: Техніка, 2000. – 498 с.
5. Барановський В. М. Загальна фізика : курс лекцій / В. М. Барановський, О. В. Черенков– Т. 2. Київ: Європейський ун-т, 2004. – 188 с.
6. Бублик Г. Ф. Курс фізики. / Г. Ф. Бублик, В. В. Левандовський, Г. Ф. Півень. – Кн. 1: Фіз. основи механіки та електромагнетизм. – Київ: Либідь, 2001. – 448 с.
7. Король А. М. Фізика: механіка, оптика, квантова фізика : підруч. для студ. техн. ВНЗ / А. М. Король, М. В. Андріяшик. – Київ: ІНОКС, 2006. – 344 с.
8. Сусь Б. А. Коливання і хвилі: навч. посіб. / Б. А. Сусь, В. Ф. Заболотний, Н. А. Мисліцька. – Київ: КПІ ім. Ігоря Сік орського, 2021. – 112 с.
9. Козицький С. В. Курс загальної фізики / С. В. Козицький, Д. Д. Поліщук. – Т. 1: Механіка. – Одеса: Астропринт, 2011. – 471 с.
10. Палєхін В. П. Курс фізики : підручник для студентів геол.-геогр. спец. / В. П. Палєхін. – Харків: ХНУ ім. В. Н. Каразіна, 2013. – 503 с.

11. Halliday D. Fundamentals of physics / D. Halliday, R. Resnick, J. Walker. – 12th ed. New York: Wiley, 2021. – 1232 p.

12. Young H. D. University physics / H. D. Young, R. A. Freedman. – Vols. 1–3. 15th ed. Boston: Pearson, 2019. – 2064 p.

13. Kleppner D. An introduction to mechanics / D. Kleppner, R. J. Kolenkow. – Cambridge: Cambridge University Press, 2014. – 360 p.

14. Griffiths D. J. Introduction to electrodynamics / D. J. Griffiths. – 5th ed. Cambridge: Cambridge University Press, 2024. – 624 p.

15. Feynman R. P. The Feynman lectures on physics / R. P. Feynman, R. B. Leighton, M. Sands. – Vols. I–III. Pasadena: California Institute of Technology, 1964.

16. Sears F. W. University physics with modern physics / F. W. Sears, M. W. Zemansky, H. D. Young. – 14th ed. – New York: Pearson, 2017. – 1600 p.

17. Serway R. A. Physics for scientists and engineers / R. A. Serway, J. W. Jewett. – 10th ed. – Boston: Cengage Learning, 2018. – 1488 p.

18. Tipler P. A. Physics for scientists and engineers / P. A. Tipler, G. Mosca. – 7th ed. – New York: W. H. Freeman, 2014. – 1328 p.

19. Giancoli D. C. Physics: principles with applications / D. C. Giancoli. – 7th ed. – Boston: Pearson, 2013. – 1040 p.

20. Chabay R. Matter & interactions / R. Chabay, B. Sherwood. – Vol. 1. – 3rd ed. – New York: Wiley, 2015. – 648 p.

Електронне навчальне видання

БЕЗУГЛИЙ Анатолій Васильович
ШИШКО Наталія Сергіївна

ФІЗИКА
ЗАГАЛЬНА ФІЗИКА

КОНСПЕКТ ЛЕКЦІЙ

(для здобувачів першого (бакалаврського) рівня вищої освіти денної та заочної форм навчання для всіх спеціальностей)

(Англ. мовою)

Відповідальна за випуск *Г. Я. Дульфан*
За авторською редакцією
Комп’ютерне верстання *Н. С. Шишко*

План 2025, поз. 163Л

Підп. до друку 26.08.2025. Формат 60 × 84/16.
Ум. друк. арк. 7,8.

Видавець і виготовлювач:
Харківський національний університет
міського господарства імені О. М. Бекетова,
вул. Чорноглазівська (Маршала Бажанова) 17, Харків, 61002.
Електронна адреса: office@kname.edu.ua
Свідоцтво суб’єкта видавничої справи:
ДК № 5328 від 11.04.2017.